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THE ASSOCIATION OF
MATHEMATICS TEACHERS OF INDIA

The Association of Mathematics Teachers of India (AMTI) was started in 1965 for the promotion of efforts to improve Mathematics education at all levels. A major aim of the Association is to assist practising teachers of Mathematics in schools in improving their expertise and professional skills. Another important aim is to spot out and foster Mathematical talents in the young. The Association also seeks to disseminate new trends in Mathematics education among parents and public. Other activities of the Association include consultancy services to schools in equipping the Mathematics section of their libraries, in organising children's Mathematics clubs and fairs, in setting up teacher centres in schools, in conducting Mathematics laboratory programmes, in holding practical tests in Mathematics in assisting children in participating investigational projects.

The Association holds " National Mathematical Talent Search Competition " annually and organizes Orientation Courses, Seminars and Workshops for teachers and courses for talented students. A national conference is held annually in different parts of the country for teachers to meet and deliberate on important issues of Mathematics education. Innovative teacher award has been instituted to give public recognition to enterprising and pioneering teachers of Mathematics for which entries from teachers are invited.

. An award for contributions to the Mathematics Teacher relating to History of Mathematics in the context of mathematics education had been instituted by Prof.R.C.Gupta.

"The Mathematics Teacher (India)" (MT) is the official quarterly journal of the Association and is issued twice a year. It has been approved for use in schools and colleges of education by the Government departments of education in many States. Besides MT the Association also brings out Junior Mathematician (JM), three issues in a year, especially for school students in English and Tamil.

The membership of the Association is open to all those interested in Mathematics and Mathematics Education. The membership fee inclusive of subscription for "**The Mathematics Teacher (India)**" and effective from April 1993 is as follows:

Subscription for India*

Category	Individual	Institutional
Life	Rs. 500	Rs. 1000
Annual (Ordinary)	Rs. 50	—
Junior Mathematician - Life	Rs. 250	Rs. 250
Junior Mathematician - Annual	Rs. 25	Rs. 25

The Journal "The Mathematics Teacher" will be supplied free to all members.
Fifty or more subscriptions to Junior Mathematician will entail 20% discount.

** For countries other than India same figures in US $.*
(Inclusive of postage)
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Dr. S.Muralidharan
Editor

EDITORIAL

Success in a profession is not possible without continuous learning. In this Internet age, there are innumerable options for learning from the comfort of ones home. But from time to time, one should also meet fellow professionals to compare notes, learn techniques that have succeeded and just to keep in touch. Professional conference is the best forum for meeting with teachers from other countries. In this issue, we publish a report on the ICME 13 held in the city of Hamburg from July 24 to 31, 2016. Dr Agarkar's report gives a vivid account of the conference proceedings.

We all *know* that our subject is beautiful. When we see a beautiful proof or a solution, we feel it is poetry in Mathematics. It is our duty to teach our students the art of appreciating good Mathematics. The paper "Beautiful Mathematics" gives some beautiful theorems and solutions. I am sure, you have your own set of mathematical poetry. You are welcome to share them with other readers through this journal.

We are happy to publish the table of contents of our journal volumes 1 to 50. We plan to publish interesting articles from our archive in the coming years.

The Annual conference of AMTI is a very popular event and we have hosted very many illustrious speakers in our conferences.

This issue includes the Bhatnagar Memorial lecture, R C Gupta Endowment lecture, Narasinga Rao memorial lecture and Presidential address delivered at our 50th Conference.

A report on association activities has been included. We are happy to note we have a new President for our association. It is heartening to observe that the popularity of NMTC is steadily increasing.

Professor K S Ramachandran was a well known Mathematics teacher. He passed away on July 2nd. An obituary appears in this issue.

As usual next issue of the journal will be dedicated to NMTC solutions.

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ICME 13: A Grand Mela of Mathematics Educators

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Prologue

About five decades ago some Mathematics Educators came together and established International Council of Mathematical Instruction (ICMI). The Executive Body of ICMI decided to bring together mathematics educators from different parts of the world after a fixed interval of time. Accordingly, the first International Congress on Mathematical Education (ICME 1) was held in 1969 at Lyon in France. Since then ICME is held every four years in different countries of the world. Here is a list of ICME events held so far.

- ICME-1, 1969, Lyon (France)
- ICME-2, 1972, Exeter (UK)
- ICME-3, 1976, Karlsruhe (Germany)
- ICME-4, 1980, Berkeley (USA)
- ICME-5, 1984, Adelaide (Australia)
- ICME-6, 1988, Budapest (Hungary)
- ICME-7, 1992, Qubec (Canada)
- ICME-8, 1996, Sevilla (Spain)

- ICME-9, 2000, Makuhari (Japan)
- ICME-10, 2004, Copenhagen (Denmark)
- ICME-11, 2008, Monterrey (Mexico)
- ICME-12, 2012, Seoul (Korea)

ICME-13 (Thirteenth International Congress on Mathematical Education) was held in the City of Hamburg from July 24 to 31, 2016. It was attended by about 4000 academicians from 109 countries. They were from different areas like pure mathematics, applied mathematics, mathematics education, history of mathematics, etc. Everyone had something to share based on his/her experience. For the whole week the entire campus of the Hamburg University, the venue of the congress, was full of mathematical activities. During the inaugural session it was informed that Germany was hosting ICME event for the second time. It had hosted ICME 3 at Karlsruhe forty years ago that is 1976. This time they had chosen a peaceful city of Hamburg located in the northern part of the country. It is a popular tourist destination and has good educational institutions. I had an opportunity to participate in this congress. This article is based on the rich experiences gained during the congress.

Academic Programme

The academic programme of the Congress was announced more than a year ago. The deadlines given for submission of proposals were strictly adhered to. All the submissions were through Internet system using Conftool. Each proposals was peer reviewed. Review reports were sent to concerned

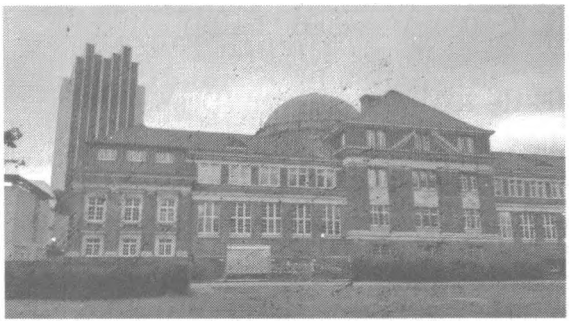

Figure 1: Main building of Hamburg University

person for the modification of his/her proposal. The number of proposals received were very large in number. Hence the academic programme of the Congress was divided into several parts:

A Plenary Activities

B Thematic discussions

C Invited lectures

D ICMI Studies and surveys

E Presentations by ICMI Affiliate Organizations

F Topic Study Groups

G Discussion Groups /Workshops

H Posters

I Exhibition

J Early Career Researchers Day

K Teachers' Activities

L National Presentations

Salient features of each of the sessions are described below.

A. Plenary Activities

Plenary Activities had two components: Panel discussion and Plenary lectures.

1. Panel Discussion

Two panel discussion sessions were arranged on the first and last day of the Congress. The theme of the first session was International Comparative Studies in Mathematics. The panelists were: Professor Jinfa Cai (University of Delaware, USA), Professor Ida Mok (University of Hong Kong, China), Professor Vijay Reddy (Human Sciences Research Council, South Africa), and Professor Kaye Stacey (University of Melbourne, Australia) and was chaired by Jinfa Cai. After her introductory remarks each of the panelists made presentation in a stipulated time. At the end the chairperson summarized the discussions and presented her own views on international comparisons.

The second panel discussion was held on the last day of the Congress. Members of the panel were: Professor Ghislaine Gueudet (University of Brest, France), Professor Marian Bosch (Universitat Ramon Lull, Spain), Professor Andre dsSessa (University of California, USA) and Professor Leven Versxhaffel (Universitat of Leuveh, Belgium). The discussion was monitored by Professor Ghislaine Gueudet and the topic was Transition in Mathematics Education. The panelists expressed their opinions related to the transition of students at different stages of their lives. One member discussed the

difficulties encountered in transition from primary to secondary to tertiary levels of education. Another panelist commented on the transition of mathematical knowledge from society to school while the third panelist focused attention on the transition of formal mathematics to work place.

It must be noted that there was a tremendous coordination among all the panel members. There was neither repetition nor contradiction among them. They have been interacting with each other almost for last two years. This resulted in a coordinated effort and forceful submission of their opinions in a convincing manner.

2. Plenary Lectures

Four plenary lectures were arranged during the congress. Renowned mathematics educators were invited to deliver these speeches. Details of these sessions are given below.

i) The first plenary lecture on Mathematics Education and Culture: A Contemporary Moral Imperatives was delivered by Professor Bill Barton of the University of Auckland, New Zealand. Dr. Barton took stock of the historical developments in Ethnomathematics research and suggested ways and means to nurture the dorsal spine of mathematics education.

ii) The second plenary lecture was given by Professor Gunter M. Ziegler of the Freie Universitat, Berlin, Germany. In his impressive lecture he discussed the images of mathematics possessed by students, teachers and parents. Based on his studies he gave some concrete suggestions for the improvement of teaching school mathematics.

iii) The third plenary lecture was given by Professor

Berinderjeet Kaur of the National Institute of Education, Singapore. The title of her talk was Mathematics Classroom Studies - Multiple Windows and Perspectives. Referring to the Third International Mathematics and Science Study (TIMSS) conducted in 1995 she focused her attention on the high scores obtained by Asian students. Through her studies she tried to highlight this aspect and provide answers to related questions.

iv) The fourth plenary speech on Uncovering the Special Mathematical Practices was given by Professor Deborah Loewenberg Ball of the University of Michigan, USA. Using the videos clips taken during the classroom interaction she attempted to explain how the acquisition of mathematical skills takes place.

The duration of each plenary speech used to be one hour. Since the number of persons attending these speeches was in thousands, no questions answer session could be arranged soon after the speech. Nonetheless, a special discussion session was arranged in a separate room after the speech. All those interested to meet the plenary speaker were invited to attend this discussion session and get their doubts clarified.

B. Thematic Discussion

One full afternoon, called thematic afternoon was reserved for discussions on selected European practices in mathematics education. It was basically a panel discussion where members of the panel expressed their opinions regarding a particular theme. There were three themes for discussion arranged in parallel sessions as stated below.

Theme 1: European Didactic Traditions and the panel

members were: Werner Blum (Germany) Chair, Michle Artigue (France), Marja van den Heuvel-Panhuizen (Netherlands), Maria Alessandra Mariotti (Italy) and Rudolf Strer (Germany). Theme 2: German Speaking Traditions in Mathematics education Research and the panel members were: Hans-Niels Jahnke (Germany) Chair, Marcus Nhrenbrger and Bettina Rsken-Winter (Germany), Angelika Bikner-Ahsbahs and Andreas Vohns (Germany / Austria), Lisa Hefendehl-Hebeker and Rudolf vom Hofe (Germany), Rolf Biehler and Hans-Niels Jahnke (Germany), Uwe Gellert and Gtz Krummheuer (Germany), Timo Leuders and Andreas Schulz (Germany), Andreas Obersteiner and Kristin, Gilbert Greefrath and Katrin Vorhlter (Germany), Theme 3: The Legacy of Felix and the participants were Hans-Georg Weigand (Germany) Chair, William McCallum (USA), Marta Menghini (Italy), Michael Neubrand (Germany) and Gert Schubring (Germany).

C. Invited lectures

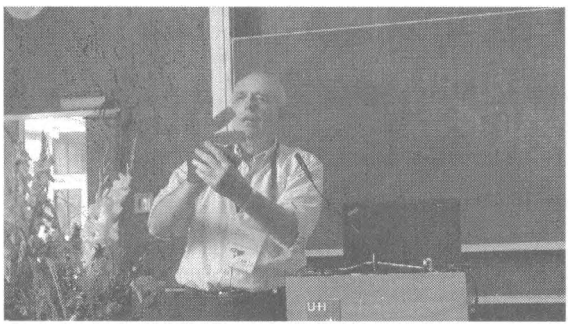

Figure 2: An invited lecture on Pedagogy

Academicians in mathematics and mathematics educators from

different countries were identified and invited to deliver a lecture during the congress. These lectures were divided into five slots of one hour each. Each slot had on an average twelve lectures arranged at twelve different places in the campus. The conference booklet given to each participant showed the places and also provided guidelines to reach there. Each participant was expected to decide which lecture to attend and how to reach the place. Enough free time was given to them to reach the place they wanted. A variety of topics starting from classroom interaction to learning theories were covered though through these lectures.

D. ICMI Studies and Surveys

ICMI has a practice of sponsoring research studies. These studies are assigned to a group of mathematics educators from different countries. Three completed studies were presented during the congress.

i) ICMI study 21 on Mathematics Education and Language Diversity

ii) ICMI study 22 on Task Design

iii) ICMI study 23 on Primary Mathematics Study on Whole Numbers.

In addition to sponsored studies ICMI also supports conducting surveys. Of these, following five surveys were presented during ICME 13.

i) Distance Learning, Blended Learning, e-learning in Mathematics.

ii) Teachers Working and Learning through Collaboration

iii) Conceptualization of the Role of Competencies, Knowing and Knowledge in Mathematics Education Research.

iv) Geometry Education including Technology

v) Survey on Assistance of Students with Mathematical Learning Difficulties - How can Research Support Practice?

E. Presentations by ICMI Affiliate Organizations

ICMI is a council of many international organizations of mathematics education. Some of these organizations are requested to make their presentations during the congress. During ICME 13 the following organizations made presentations.

i) Inter American Committee on Mathematics Education (CIAEM)

ii) Commission Internationale Pour L'etude et L'amelioration de L'enseignement des Mathematiques (CIEAEM).

iii) European Society for Research in Mathematics Education (ERME).

iv) Young Researcher in Mathematics Education (YERME).

v) International Study Group on the Relations between History and Pedagogy of Mathematics (HPM).

vi) International Study Group for Mathematical Modeling and Applications (ICTMA).

vii) International Organization of Women and Mathematics Education (IOWME).

viii) International Group for Mathematical Creativity and Giftedness (MCG).

ix) Mathematics Education Research Group of Australasia (MERGA).

x) International Group for the Psychology of Mathematics Education (PME).

xi) World Federation of National Mathematics Competitions (WFNMC).

F. Topic Study Groups (TSG)

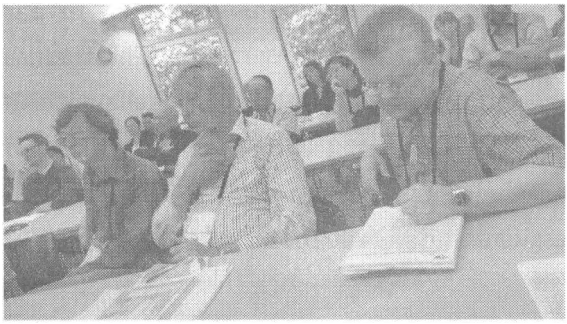

Figure 3: Discussion in TSG 35 on Ethnomathematics

As the number of participants in the ICME is quite large the organizers have decided to arrange discussion under different Topic Study Groups (TSG). There were 45 TSGs in ICME 13. Chief organizer and Associate organizers were identified for each TSG much in advance. These topics were made known at the time of conference announcement itself. Mathematics educators were advised to submit their proposals under one of these TSGs. It is expected that all the members of the group remain connected to their TSG throughout the congress. Presentations made under each TSG are compiled and brought

out as publications.

G. Discussion Groups/Workshops

Discussion groups were designed to gather together participants interested in discussing certain challenging, controversial or emerging issues and dilemmas in mathematics education. A variety of themes for discussion groups were declared prior to the conference and participants were appealed to submit the proposal in one of them. During the congress there were about 40 discussion groups. For each discussion group a submitting organizer was identified. Anyone interested in participating was expected to write to the organizer. The themes for these groups varied from misconceptions in mathematics to teacher professional development. There were two 90 minutes slots reserved for the discussion groups. They had to be arranged in parallel sessions in different rooms.

Figure 4: A workshop on Using Technology in Mathematics Education

In addition to discussion groups ICME also encourages the organization of workshops for the participants. Of the large number of workshops proposals submitted to the Local

Organizing Committee, about 40 workshops were accepted. A time slot of 90 minutes was reserved for the workshops arranged at different places in parallel sessions. The conference booklet gives a short summary of each workshop. Participants are expected to go through the abstract and decide which workshop to attend. These workshops enabled to achieve a close interaction between the workshops organizers and the audience.

H. Posters

Like any other international conference the participants were encouraged to submit poster proposals to ICME organizers also. Specific spaces were reserved for poster presenters in the campus of the university. Also, a specific time slot was reserved for watching the posters by the participants. Poster presenters were expected to make themselves available during this period to explain the content of the poster. Many of these posters were linked to the themes of Topic Study Groups.

I. Exhibition

Taking the opportunity of an international event an institution of Mathematical Sciences in Germany arranged an exhibition on Mathematics Education. It contained a variety of teaching aids that can be used for the teaching of Mathematical concepts in schools.

J. Early Career Research Day

As a part of the conference a full day programme was arranged for the newcomers in Mathematics Education. The first day

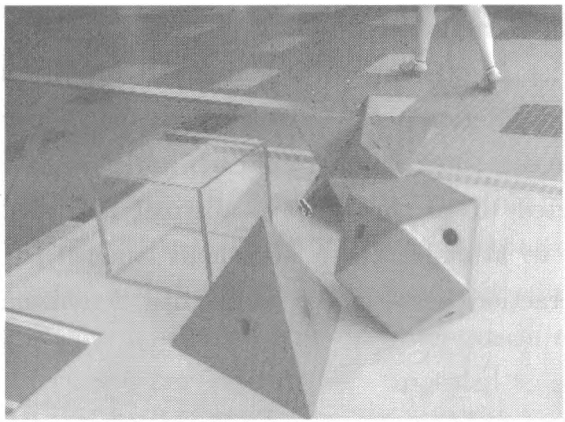

Figure 5: Some solids displayed during the exhibition

of the Congress was used for this purpose. New researchers in Mathematics Education were encouraged to register for the programme for a small charge. Experts in conducting research in Mathematics Education were invited to guide the new entrants. A large number of research scholars and practicing teachers took the benefit of these sessions.

K. Teacher's Activities

It is well known that the teachers play a major role in transmitting mathematics curriculum to the students. It is, therefore, necessary that practicing teachers be made aware of new research in Mathematics Education. Some of the participants of the Congress were practicing teachers themselves. In addition, special sessions were arranged for German teachers. For their convenience the discussion was arranged in German. Many of the teachers whom I met showed satisfaction on the inputs that they received during the sessions. These sessions enabled them to share new methods that they

have practiced and learn more methods from others.

L. National Presentations

As a part of the Congress some selected countries are requested to make presentations on the status of mathematics education in their respective countries. Presentation based on Mathematics Education in India was made in ICME 12 held at Seoul in Korea four years ago. For ICME 13 the countries selected were Argentina, Brazil, Ireland, Japan, Turkey, and Lower Mekong Region. These presentations were arranged in parallel sessions in different buildings at the same time. As a result one could not attend more than one national presentations at a time. However, a short summary of each presentation is given in the conference booklet. The cursory glance through these summaries enable us to learn how mathematics education is tackled differently in different countries.

Epilogue

After a week long academic interaction all the participants of ICME 13 returned with rich experiences. It was a point of satisfaction that appreciable number of Indian participants attended the event. For the first time the presence of the Association of Mathematics Teachers of India (AMTI) was felt. Two active members of AMTI namely Shri S. R. Santhanam and Shri Athmaraman attended the conference full time. As a result, a large number of participants of the Congress came to know about the activities of AMTI. It would be nice if AMTI comes forward to host one of the future events in India.

Apart from good academic programme the administrative

arrangements made during the congress were also satisfactory. Prof Gabriele Kaiser of the University of Hamburg was the Convener of ICME 13. The University offered all the secretarial support required in the organization of this mega event. Professor Rudolf vom Hofe, President, Society of Didadics of Mathematics was the co-convener. The entire machinery of the society came to help in the organization of the Congress. In addition, the German Mathematical Society, the German Research Association and the German Association for the Advancement of Mathematics and Science Education offered their support to make the event a success. It is notable that Municipal Council of the city of Hamburg also offered its help in the organization of the conference. In fact, the mayor hosted a dinner for the selected participants of the congress.

Figure 6: Statues near the Congress Centrum showing Final Departure

The conference ended with a grand closing ceremony. It was basically a programme to recognize the efforts made by different persons in making the event a success. It was followed

by a delightful musical performance by the local artists. A memorable part of the closing ceremony was the invitation given by Chinese delegation to attend ICME 14 to be held in Shanghai from July 12 to 19, 2020. We should prepare to participate in this event in large numbers.

Beautiful Mathematics

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In his book "A Mathematician's Apology", Professor Hardy says *The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.*

When mathematicians describe equations as beautiful, they are not lying. Brain scans show that their minds respond to beautiful equations in the same way other people respond to great paintings or masterful music [12]. Many times we see a charming proof or a beautiful solution that gives us immense joy. It also makes us wonder how the author could have got *that* idea. The book [1] collects several charming proofs. Again, in [2], several beautiful geometrical arguments are given – many of these are elegantly intuitive. The first step towards teaching Mathematics is to show the students that the subject has immense beauty. We need to tell them how to enjoy that beauty. Towards this objective, I present a set of charming proofs and beautiful solutions that I liked most. All of these are within the reach of a school student.

1. **Sum of an arithmetic progression**

   A common anecdote about Gauss involves his school teacher who wanted to take a rest and asked the students to sum the integers from 1 to 100 as busy work. After

a few seconds, the teacher saw Gauss sitting idle. When asked why he was not frantically doing addition, Gauss quickly replied that the sum was 5050. His classmates and teacher were astonished, and Gauss ended up being the only pupil to calculate the correct answer. How could young Gauss have added 100 numbers? A simple but charming trick that perhaps Gauss found was this "reverse and add":

$$S = 1 + 2 + 3 + \cdots + 100$$
$$S = 100 + 99 + 98 + \cdots + 1$$

If we add vertically, we get 101 in each column and we have 100 such 101s. Thus $2S = 100 \times 101$ and hence $S = 5050$. This technique works for a general arithmetic progression as well:

$$S = a + (a + d) + (a + 2d) + \cdots + (a + (n - 1)d)$$
$$S = (a + (n - 1)d) + (a + (n - 2)d) + \cdots + a$$

Adding vertically, we obtain

$$2S = (2a + (n - 1)d) + (2a + (n - 1)d) + \cdots + (2a + (n - 1)d)$$

and hence $S = \dfrac{n(2a + (n - 1)d)}{2}$.

Since the binomial coefficients satisfy the property,

$$\binom{n}{i} = \binom{n}{n - i}$$

this technique of reverse and add works for sum of binomial coefficients multiplied by terms of an arithmetic

progression. Thus

$$a\binom{n}{0} + (a+d)\binom{n}{1} + \cdots + (a+nd)\binom{n}{n}$$
$$= \frac{1}{2}(a + (a+nd))2^n$$
$$= 2^{n-1}(2a+nd)$$

2. **Tangent Sweep and Pythagoras Theorem**

There are numerous proofs for Pythagoras theorem – possibly the most for any mathematical theorem. The following proof from [2] is really a charming one:

Let a, b are the sides of a right angled triangle with hypotenuse c. Draw two concentric circles of radii a, c and consider the area of the annular region formed between them. This is clearly $\pi c^2 - \pi a^2$.

Now consider the tangents to the inner circle. The

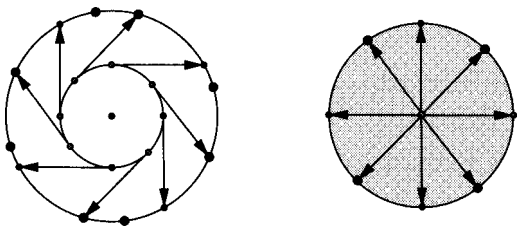

Figure 1: Tangent Sweep

segment of these tangents intercepted between the circles sweep the area of the annulus. If we translate these tangents to a common point, they sweep a circle with that point as center and radius equal to the length of

these tangents. Clearly, these tangents have length b and hence the area of the annulus is the same as the area of a circle of radius b. Thus we have

$$\pi(c^2 - a^2) = \pi b^2$$

and $c^2 = a^2 + b^2$.

Tangent sweep is a very intuitive idea and Mamikon exploits it to prove non-trivial theorems that otherwise require messy integration. For example, consider the region described between an ellipse and the tip of a moving tangent of constant length (Such a curve is traced by the front wheel of a bicycle when the rear wheel moves in an ellipse). Again when we translate the tangent

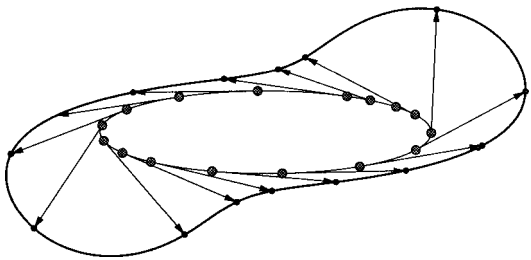

Figure 2: Cycle Track

segments to a common point, they describe a circle with radius equal to the length of the segment and hence the area between these curves is $\pi\ell^2$ where ℓ is the length of the tangent segment. This intuitive proof is very appealing compared to a proof using integration (see [10]). For more examples of the use of this beautiful

technique, see [2].

3. **Square within a square**
   Create a square within a square by drawing a line from the midpoint of each side to one of the opposite vertices, as shown in Figure 3. Then the area of the smaller square is one fifth of the area of the larger. A charming visual proof is given by the picture on the right. Visual proofs

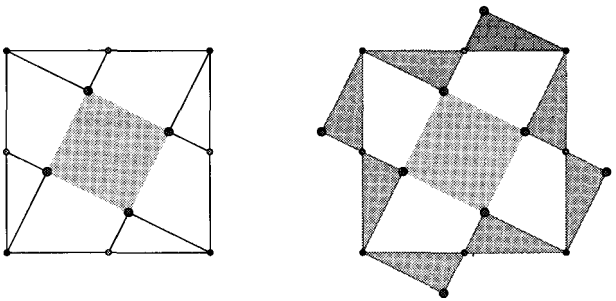

Figure 3: Square within a square

   are very appealing to students. The series of books [6], [7], [8] contain several visual thinking ideas.

4. **Euler's Formula for Polyhedra** $V - E + F = 2$
   A 1988 poll of readers of the *Mathematical Intelligencer* magazine ranked some of the most well-known theorems in mathematics. The theorem ranked as second was the Euler's Formula for Polyhedra. An interesting account of the history and its deeper connections to Topology of surfaces, see [9]. Simple though it may look, this little

formula encapsulates a fundamental property of those three-dimensional solids we call polyhedra, which have fascinated mathematicians for over 4000 years.

A polyhedron is a closed, solid object whose surface is made up of a number of polygons. We call these polygons *faces* of the polyhedron. Any two of the faces must intersect along sides of the respective polygons and we call these intersections *edges*. Again two edges must intersect at a vertex of the respective polygons and these points are called the *vertices*. We will assume that our polyhedra

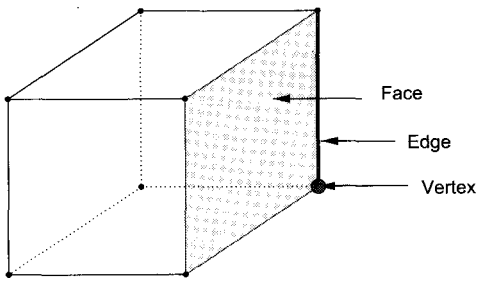

Figure 4: Vertex, Edge and Face of a Polyhedron

are in *one piece* and *convex*. In other words, we must be able to deform the polyhedra (without tearing any of its faces) so that it becomes a sphere. Some examples of polyhedra are shown in Figure 5. Let us count the number of vertices, edges and faces:

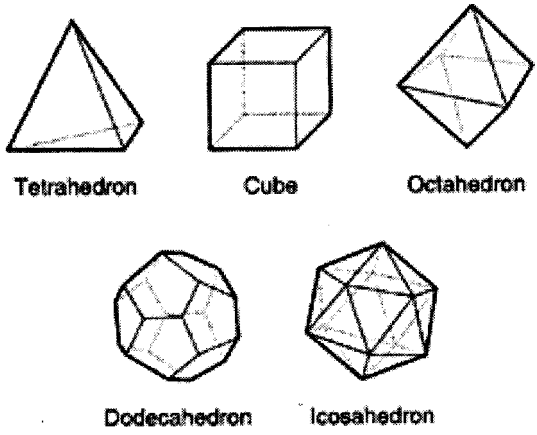

Figure 5: Examples of Polyhedra

Polyhedra	Vertices	Edges	Faces	$V - E + F$
Tetrahedron	4	6	4	2
Cube	8	12	6	2
Octahedraon	6	12	8	2
Dodecahedron	20	30	12	2
Icosahedron	12	30	20	2

In all of these, we find that $V - E + F = 2$.

The charming proof of this is as follows:

Holding the polyhedron with one face pointing upwards, let us remove this face but leave its edges and vertices behind. Now assuming that the polyhedron is made up of flexible material, we can flatten the polyhedron to form a network of points and lines. If we do this for a cube we obtain the network shown in Figure 6. Since we have removed one of the faces, the quantity $V - E + F$ for this net is one less than that for the polyhedron. Thus we need to prove $V - E + F = 1$ for this net.

Step 1 We look for polygonal faces that have more than

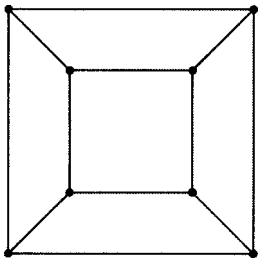

Figure 6

three sides. We *triangulate* each such face by drawing non intersecting diagonals. For the cube net, we have illustrated the step in Figure 7. When we draw each such

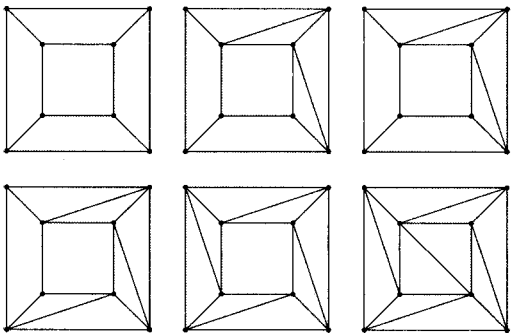

Figure 7: Triangulated cube net

diagonal, we obtain one more face and one more edge. The number of vertices do not change. Thus V remains the same, E becomes $E+1$ and F becomes $F+1$. Hence $V-E+F$ becomes $V-(E+1)+(F+1)$ and remains invariant.

Step 2 We now check if there are any faces that share

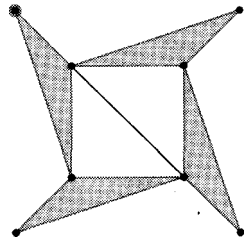

Figure 8: After Step 2

only one edge with the exterior boundary. We remove this face by removing the one shared edge but keep the end points of the edge in the net. The face merges with the exterior boundary, and hence F becomes $F-1$, E becomes $E-1$ but V does not change. Again $V-E+F$ does not change.

Step 3 Now we check if there are faces that share two edges with the boundary. If there is such a face, we remove both the edges and the shared vertex between these edges. This removes one vertex, two edges and one face. Thus $V-E+F$ becomes $(V-1)-(E-2)+(F-1)$ and hence does not change.

Now that we know that neither of Steps 2 and 3 change $V-E+F$, we perform these operations till we get a simple net – a triangle. For this net, $V=3, E=3, F=1$ and hence $V-E+F=1$ and the proof is complete.

There are two important rules to follow when doing this. Firstly, we must always perform Step 3 when it is possible to do so; if there is a choice between Step 2 and Step 3 we must always choose Step 3. If we do not, the network may break up into separate pieces. Secondly, we must

only remove faces one at a time. If we do not, we may end up with edges sticking out on their own into the exterior face, and we will no longer have a proper network. Also note that the process does terminate, since at each step we reduce the number of faces and the net for any polyhedron has only a finite number of faces.

Euler's formula has several applications. We give one of these about the number of pentagons in a golf ball. A

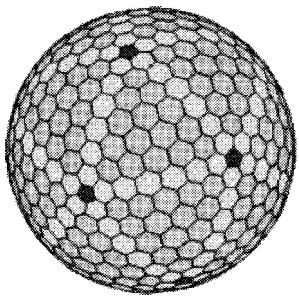

Figure 9: Golf ball

typical golf ball is a polyhedron made up of hexagons and pentagons such that at each vertex three polygons meet. If F_6 is the number of hexagons and F_5 is the number of pentagons, then we have

$$F = F_6 + F_5, \quad 2E = 5F_5 + 6F_6, \quad 3V = 5F_5 + 6F_6$$

Hence

$$\begin{aligned} 2 &= V - E + F \\ &= \frac{5F_5 + 6F_6}{3} - \frac{5F_5 + 6F_6}{2} + F_6 + F_5 \\ &= \frac{F_5}{6} \end{aligned}$$

Thus $F_5 = 12$. The number of hexagonal faces can be any number ≥ 2, but the number of pentagons is always 12. A cool fact, indeed.

5. **Shortest Paths**

The shortest path between two points in a plane is the straight line joining those points. However, on other surfaces, shortest paths (also called *geodesics*) between points is not easy. For example, consider a conical glass (Figure 10). On the top rim, at the point M, there is an ant. On the diagonally opposite point N on the rim, there is a drop of honey. What is the shortest path ant can take to reach the honey drop?

If we cut the glass along AN and spread it out, we see that the shortest path is not along the top rim of the glass. Sometimes, we need to cut the surface carefully to

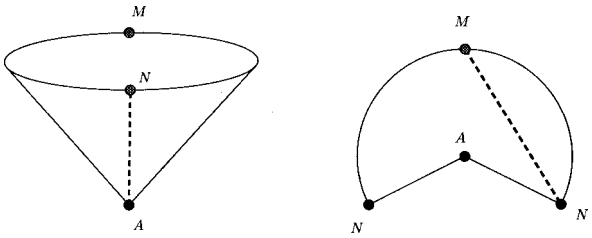

Figure 10: Shortest path in a conical glass

get the shortest path. Consider the following example: $ABCDEFGH$ is a cube of side length a. On the mid point M of the side AD there is an ant. It wants to reach a food particle at the point N located at a distance $\frac{a}{4}$ away from G on the side FG. What is the shortest

distance that this ant can travel to arrive at N?

We can open up the cube as a net to find the shortest

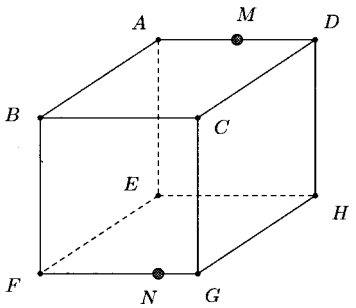

Figure 11: Cube

path. Perhaps, we may think that the path crossing minimum number of faces is shortest. Hence if we open the cube as in Figure 12, and drop a perpendicular NP from N on AD, then in the right angled tr we obtain the triangle MNP, $MP = \frac{a}{4}$ and $NP = 2a$. Thus the distance MN is

$$a\sqrt{\frac{1}{16} + 4} = a\frac{\sqrt{65}}{4}$$

But, surprisingly, the shortest path is through the face $CDHG$. Open the cube as in Figure 13. Now, in the right angled triangle MCN, $MC = \frac{3a}{2}$ and $CN = \frac{5a}{4}$. The length of this path is

$$a\sqrt{\frac{9}{4} + \frac{25}{16}} = a\frac{\sqrt{61}}{4}$$

This path is through the faces $AEDH$, $CDHG$ and $FEHG$. We can also open up as in Figure 14 to get

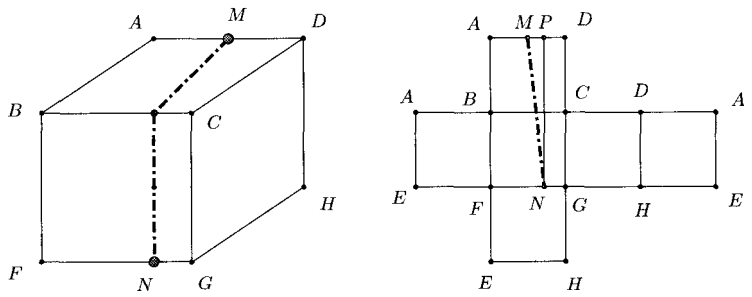

Figure 12: A shorter path

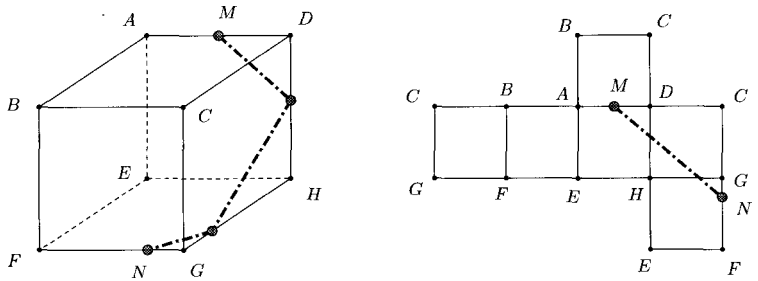

Figure 13: Shortest path

an equal length path through the faces $ADCB$, $CDHG$ and $BCGF$.

6. **"Seven is More Than Six. Pigeon-Hole Principle"** This is the title of the first chapter in the book [3]. Even though pigeon-hole principle is easy to state and understand, very many deeper proofs have been given using this principle. The beauty in such proofs is

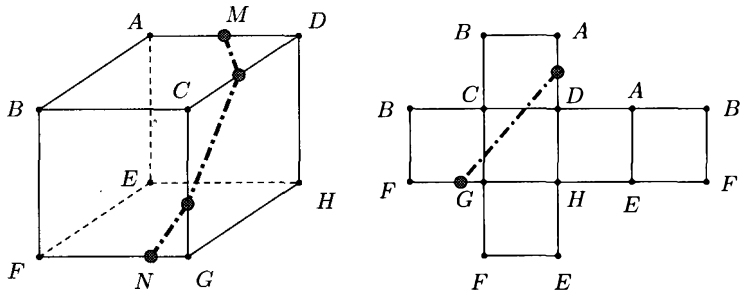

Figure 14: Another shortest path

identifying the "pigeons" and the corresponding "holes" to put them in! The following example from [11] illustrates this:

Forty one rooks are placed in a 10×10 chessboard. Prove that you can choose five that do not attack each other. We will see two proofs.

First proof Since 41 rooks (pigeons) are placed on ten rows of the board (pigeonholes) and $41 = 4 \times 10 + 1$, there exists a row A with at least 5 rooks on it. Let us remove row A; we now have nine rows (pigeonholes) left with at least $41 - 10 = 31$ rooks (pigeons) on them. Since $31 > 3 \times 9 + 1$, there is a row B among the nine rows with at least 4 rooks on it. Now we remove row B. We are left with eight rows (pigeonholes) and at least $41 - 2 \times 10 = 21$ rooks (pigeons) on them. Since $21 = 2 \times 8 + 1$, there is a row C among the eight rows with at least 3 rooks on it. Continuing this reasoning, we get row D with at least 2 rooks on it and row E with at least 1 rook on it.

Now we are ready to select the required five rooks. First we pick any rook R_1 from row E. At least one exists there. Next, we pick a rook R_2 from row D, which is not in the same column as R_1. This can be done, too, because at least two rooks exist in row D. Next, of course, we pick a rook R_3 from row C, which is not in the same column as R_1 and R_2. Conveniently, this can be done, since row C contains at least three rooks. Continuing this construction, we end up with five rooks $R_1, R_2, \ldots, R_5$ that are from different rows (one rook per row out of the selected rows A, B, C, D, E) and different columns; therefore, they do not attack each other.

Now, the ***colorful* proof**:

Glue together two opposite sides of the chess board to

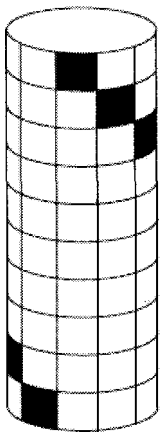

Figure 15: Coloring the diagonals

make a cylinder. Color the cylinder diagonally with ten colors. Now we have $41 = 1 + 4 \times 10$ pigeons and 10 pigeon-holes, the ten colored diagonals. Hence one of the

holes must contain at least five pigeons. Thus there are five rooks on a diagonal and hence they do not attack each other. Pigeons are interesting!

Can you recognize this *avatar* of the above problem?

Given 41 distinct two-digit numbers, prove that you can choose five numbers out of them such that any two of the five numbers have distinct units digits and distinct tens digits.

7. **Trick shots**

A single ball rests calmly on a square-shaped billiard table. The ball is hit and it moves on the table for an infinite amount of time, obeying an elementary law of physics: The angles α and β (Figure 16) are always equal. Under what circumstances will the movement of the ball be cyclic? (See [5]). We can try to introduce a

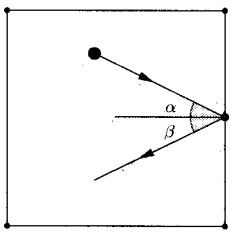

Figure 16: Trick Shot

system of coordinates in the table and try to write down the equations of the reflected paths of the ball when it

hits the sides of the board and derive the conditions under which the ball returns to the starting position. This may lead to a messy calculation since the number of reflections can be very large.

Instead of imagining the ball to reflect off of each side of the billiard table, imagine the ball as if it is traveling in a straight line and consider *reflection of the table!* (Figure 17) Each time the ball strikes a wall of the table, we

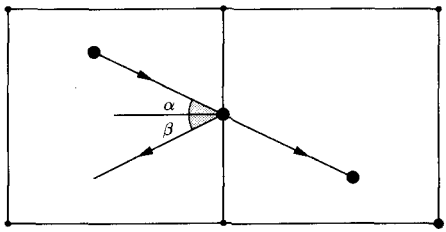

Figure 17: Reflection of the billiard table

can reflect the table on that wall. The ball will cycle, if after some number of reflections, it returns to its original position. When the ball returns to its original position, it would have traveled p tables horizontally and q tables vertically. Hence if α is the angle made by the path to the horizontal, then $\tan\alpha = \frac{q}{p}$. Clearly, when $\alpha = 0$ or $\alpha = \frac{\pi}{2}$, the ball clearly cycles.

When $\tan\alpha$ is not rational, assuming zero friction on the table and all rebounds perfect, the ball will keep moving for ever and cover all the points on the table (*space filling, indeed!*)

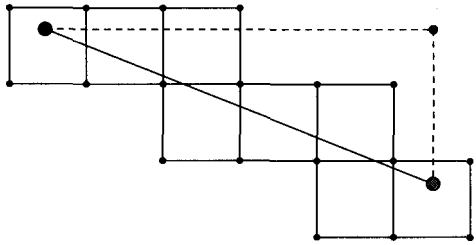

Figure 18: Getting a cycle

The trick we emplyed to solve the problem is to make the billiard tables *move* instead of the billiard balls!

8. **Counting "Stars and Bars"**
Discrete mathematics is a rich source of charming proofs. One of the counting techniques called "stars and bars" is helpful in solving a variety of so called occupancy problems. These involve counting the number of ways of placing objects in boxes. Consider the following two problems:

   (a) In how many ways can we distribute 15 identical pencils to 5 children so that each child gets at least one pencil?

   (b) In British national lottery 6 numbers are chosen from the numbers 1 to 49. Many people believe that the chance that two consecutive numbers will be chosen is extremely small. Is this true?

The first problem is equivalent to finding the number of solutions in positive integers to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 15$$

If the number on the right hand side is smaller, we could try a brute force enumeration. But the stars and bars technique solves all such problems with (almost) no computation!

We place 15 stars in a row: Now we choose four of the

Figure 19: Stars...

14 gaps between the stars and place bars: Now we let

Figure 20: ...and bars

x_1 to be the number of stars before the first bar, x_2 to be the number of stars between the first and second bars and so on and x_5 is the number of stars after the fourth bar. For example, in the Figure 20 the corresponding solution to the placement of bars is $(3, 3, 3, 2, 4)$. Clearly there is a one to one correspondence between the different placement of the bars and the solutions to the given equation. Since the number of ways of choosing 4 out of 14 gaps is $\binom{14}{4}$, the number of solutions to the equation is also $\binom{14}{4}$. Cute, isn't it?

More generally, the number of solutions in positive integers to the equation $x_1 + x_2 + \cdots + x_k = n$ is $\binom{n-1}{k-1}$

What about the second problem? We first find how many

ways can we select 6 integers from $1, 2, \ldots, 49$ such that no two are consecutive. This is stars and bars in reverse! Suppose that we have chosen the six integers such that no two are consecutive. How do we introduce them back into the set? These go into 6 of the gaps between the remaining 43 integers. Note that we now need to include the place before the numbers and also the one after all the numbers. Thus there are 44 possible places for these stars to go back in. Hence the number of ways is $\binom{44}{6}$. The probability that there are no consecutive numbers in a set of 6 integers chosen from $1, 2, \ldots, 49$ is therefore

$$\begin{aligned}\frac{\binom{44}{6}}{\binom{49}{6}} &= \frac{44!}{6!38!} \Big/ \frac{49!}{6!43!} \\ &= \frac{44 \times 43 \times \times 42 \times 41 \times 40 \times 39}{49 \times 48 \times \times 47 \times 46 \times 45 \times 44} \\ &= \frac{22919}{45402} \\ &= 0.505\end{aligned}$$

Thus the probability that there are consecutive numbers in a draw of 6 numbers is $1 - 0.505 = 0.495$, and hence not very small.

9. **Evaluating a determinant**

The following is a well-known determinant evaluation problem:

Prove that

$$A = \begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$

The usual proofs use the properties of determinants. Almost all the proofs I have seen try to make as many

elements equal to zero so that evaluation becomes easier. One such proof is the following:

To get $1 + a^2 + b^2$ in the first row, we can apply $R_1 \to R_1 + bR_3$. This will give

$$A = \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Taking $1 + a^2 + b^2$ out from the first row, we get

$$A = (1+a^2+b^2)\begin{vmatrix} 1 & 0 & -b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

We need one more zero in the first row. This can be obtained by taking $C_3 \to C_3 + bC_1$. This gives

$$A = (1+a^2+b^2)\begin{vmatrix} 1 & 0 & 0 \\ 2ab & 1-a^2+b^2 & 2a(1+b^2) \\ 2b & -2a & 1-a^2+b^2 \end{vmatrix}$$

Now, expanding by the first row, we obtain

$$\begin{aligned} A &= (1+a^2+b^2)[(1-a^2+b^2)^2 + 4a^2(1+b^2)] \\ &= (1+a^2+b^2)[(1+b^2-a^2)^2 + 4a^2(1+b^2)] \\ &= (1+a^2+b^2)[(1+b^2)^2 + a^4 - 2a^2(1+b^2) + 4a^2(1+b^2)] \\ &\qquad\qquad \text{expanding } ((1+b^2)-a^2)^2 \\ &= (1+a^2+b^2[(1+b^2)^2 + a^4 + 2a^2(1+b^2)] \\ &= (1+a^2+b^2)(1+a^2+b^2)^2 \\ &= (1+a^2+b^2)^3 \end{aligned}$$

However, a simple observation gives a proof that requires

almost no computation. Consider the matrix

$$M = \begin{pmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{pmatrix}$$

If M' is the transpose of M, it is easy to see that

$$MM' = (1+a^2+b^2)^2 I$$

and hence it follows that

$$\det(M)\det(M') = (1+a^2+b^2)^6$$

Since $\det(M) = \det(M')$, we have

$$\det(M) = (1+a^2+b^2)^3$$

Done. What a beauty! The key is in observing that the rows are pairwise orthogonal!

Let us conclude with a classic result, the AM-GM inequality:

10. **The inequality of the Arithmetic-Geometric means**

Most of the induction proofs are dull. But Cauchy's proof of the famous Arithmetic Mean - Geometric Mean inequality is a classic example of how to make an induction interesting.

Given positive real numbers $a_1, a_2, \ldots, a_n$, their Arithmetic mean is defined by

$$A = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

and their Geometric mean by

$$G = (a_1 a_2 \cdots a_n)^{\frac{1}{n}}$$

The required inequality to be proved is $G \leq A$. Of course, in the usual induction, one can start the induction for $n = 1$, where it trivially holds and assume for $n - 1$ to deduce the inequality for n. But Cauchy's proof is a charming one – consisting of a "forward" and a "backward" step: Assuming the truth for n we deduce the inequality for $2n$. This is the forward step. Then assuming the inequality for n we deduce it for $n - 1$ and this is the backward step. Starting the induction at $n = 2$, successive application of forward step proves the inequality for all powers of 2. To prove it for a general n, find a k such that $n \leq 2^k$. Since the inequality holds for 2^k, applying the backward step successively we reach the inequality for n.

To start the induction, we prove the inequality for $n = 2$. Here, we have

$$(\sqrt{a_1} - \sqrt{a_2})^2 \geq 0$$

gives

$$\frac{a_1 + a_2}{2} \geq \sqrt{a_1 a_2}$$

Assume the inequality for n. Let $a_1, a_2, \ldots, a_{2n}$ be $2n$ positive real numbers. We have

$$A_1 = \frac{a_1 + a_2 + \cdots + a_n}{n} \geq (a_1 a_2 \cdots a_n)^{\frac{1}{n}} = G_1$$

$$A_2 = \frac{a_{n+1} + a_{n+2} + \cdots + a_{2n}}{n} \geq (a_{n+1} a_{n+2} \cdots a_{2n})^{\frac{1}{n}} = G_2$$

Now, applying the inequality for G_1, G_2, we get

$$\frac{A_1 + A_2}{2} \geq \frac{G_1 + G_2}{2} \geq \sqrt{G_1 G_2}$$

Substituting for A_1, A_2, G_1, G_2 yields

$$\frac{a_1 + a_2 + \cdots + a_{2n}}{2n} \geq (a_1 a_2 \cdots a_{2n})^{\frac{1}{2n}}$$

Now for the proof of the backward step.

Assuming that the inequality holds for n, let $a_1, a_2, \ldots, a_{n-1}$ be $n-1$ positive real numbers. Let $g = (a_1 a_2 \cdots a_{n-1})^{\frac{1}{n-1}}$, the geometric mean of the a_i s. Applying the inequality for $a_1, a_2, \ldots, a_{n-1}, g$, we get

$$\begin{aligned} a_1 + a_2 + \cdots + a_{n-1} + g &\geq n(a_1 a_2 \cdots a_{n-1} g)^{\frac{1}{n}} \\ &= n(g^{n-1} g)^{\frac{1}{n}} \\ &= ng \end{aligned}$$

Thus

$$a_1 + a_2 + \cdots + a_{n-1} \geq (n-1)g$$

completing the proof.

Although Cauchy's proof is charming, the following clever proof of the inequality is equally beautiful:

Let $g = (a_1 a_2 \cdots a_n)^{1/n}$ and $b_i = a_i / g$. Then

$$b_1 b_2 \cdots b_n = 1$$

and the inequality reduces to proving the following:

If $b_1 b_2, \ldots, b_n$ are positive real numbers such that $b_1 b_2 \cdots b_n = 1$ then

$$b_1 + b_2 + \cdots + b_n \geq n$$

For $n = 1$, the result trivially holds. Assume the result for $n-1$. Without loss of generality, we can assume that b_1 is the maximum of b_i and b_n is the minimum of b_i. Thus $b_1 \geq 1$ and $b_n \leq 1$. Thus

$$(b_1 - 1)(1 - b_n) \geq 0$$

and consequently

$$b_1 + b_n \geq b_1 b_n + 1$$

Now, since

$$b_2 \times b_3 \times \cdots \times b_{n-1} \times (b_1 b_n) = 1$$

by induction hypothesis,

$$\begin{aligned} & n - 1 \leq b_2 + b_3 + \cdots + b_{n-1} + (b_1 b_n) \\ \Rightarrow \quad & n \leq b_2 + b_3 + \cdots + b_{n-1} + (b_1 b_n) + 1 \\ & \leq b_2 + b_3 + \cdots + b_{n-1} + (b_1 + b_n) \end{aligned}$$

completing the induction.

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Cultivating Mathematical Giftedness and Creativity in a Multicultural Society The Kidumatica Experience[1]

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In Hebrew, the name "Kidumatica" is a play on words, combining "kidum", meaning promotion and development, with "matematica" mathematics. In English the name has meaning too, since Kidumatica is a mathematics club for kids. The "Kidumatica mathematics club for creativity and excellence" was designed for the promotion of giftedness and creativity amongst students who come from a particularly diverse array of cultural and socio-economic backgrounds. The project and its success stand as an ongoing testament to the possibility of overcoming social and cultural obstacles, traversing barriers formed by language and gaps in schooling. Kidumatica seeks out "promising students" and helps them discover the heights of their talent and creativity. The Kidumatica model is proof that this can be done. This paper, like Kidumatica itself, is non-routine. Rather than proceeding diligently through section on theoretical background, methodology, results and conclusions, I will present that narrative of the Kidumatica math club, using examples of the non-routine problems through which our students are taught to illustrate how such tasks promote giftedness and creativity in a multi-cultural environment.

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[1]Prof. P.L. Bhatnagar Memorial Lecture
50th Annual Conference
Association of Mathematics Teachers of India (AMTI)

1. **Whetting their appetites: The January problem**
   Every year at the opening assembly for newly accepted members of the "Kidumatica" mathematics club, we present our prospective new students and their parents with the following problem:
   January problem A given January contains exactly 4 Mondays and 4 Fridays. On what day will January 20th fall? Explain your answer.
   This problem is, without a doubt "non-routine" ([1]). It cannot be associated with any specific chapter in the school curriculum, no "pre-made" formula can be automatically applied to solve it, and there is not even a given strategy for arriving at one. While one might assume that such a sudden confrontation with the unknown would be "frightening" for any school-educated individual, we have found that the problem functions rather as a unifying and a motivating force for the variety of populations and age groups that gather for our opening assembly. Far from balking at the strangeness of it, the audience parents and students alike are excited and eager to try and solve the problem, and to share their approach to doing so with the group. Thus the problem, and the shared experience of trying to solve it, is a bridge that unites the many cultures, ages and mathematical backgrounds from which Kidumatica draws its members. The January Problem is designed to be "user friendly". The simple language and phrasing minimizes its dependence on language, making it more accessible for

students studying in a language that is not their mother tongue. The context is very familiar (whatever their local customs, all Israeli schools use the Gregorian calendar). There are no fractions, or other such elements that can sometimes be deterrents for students with differing academic backgrounds, and the number of options is not large (no more than seven, since there are only seven days in a week). As a result (based on interviews we have conducted), this problem feels "like anyone could solve it."

This problem is an example of "creativity promoting equal-opportunity pedagogy." There are many ways to solve it, and each strategy must be "created" independently, since no strategy is provided in advance. Some draw the month of January on a calendar and "move" January first around from day to day until the problem's criteria are fulfilled. Others cut the month down to only 28 days so that each of the seven days gets exactly four instances, and then add the three extra days according to the problem's criteria. Some just go by trial and error until they reach the answer. Of course, everyone is required to support their answer, to disprove alternative answers, and to do so in a creative way using mathematical argumentation.

This problem is used in Kidumatica as a "warm-up" - a motivational initiation to the program both cognitively and affectively, particularly for those students whose hidden mathematical potential is still marred by low confidence in their own ability and by gaps in their school mathematics education. Solving this problem

requires a mathematical creativity that relies on what [1] call "mathematical resourcefulness," which is the ability to cope with non-routine open-ended mathematical situations by drawing on a variety of different resources – such as knowledge, skills and experience from several domains and subjects – to construct an appropriate conceptual tool. It therefore has the potential to promote students' giftedness (for definitions and ideas on creativity and giftedness, see [6], [3], [9], [8].

2. Unlocking their potential: Kidumatica mentors the mathematically gifted

Consider the great talents of history in their respective fields people like Mozart, Picasso, Magic Johnson – what is it that sets them apart? Raw talent alone? We think not. Had their talents not been fostered, had they not been mentored and pushed, they would not have been what they were, despite their talent. Talent alone is a necessary, but not a sufficient condition for reaching the top of one's field. To win an Olympic medal, a talented sportsman must train with a coach, and the same goes for concert pianists and – we believe – for mathematicians. Wealthy families identify their children's talents early and foster them. Children who find themselves on the social, economic or geographical sidelines, however, do not often have this luxury, and their hidden potential therefore goes untapped. The "Kidumatica" club for mathematical creativity and excellence was designed to address this very problem ([5]).

Since 1998, Kidumatica has devoted itself to identifying

and recruiting mathematically promising youth (aged 10 – 17) from Israel's economically disadvantaged – and very culturally diverse – southern region. Kidumaticas after-school activities, which take place on the campus of Ben-Gurion University, expose these students to a variety of mathematical subjects of varying content and strategy. The club's activities are meant to supplement rather than overlap with what they learn in school. Its curriculum is designed to develop sophisticated and creative mathematical thinking, to build the students' confidence and – ultimately – to equip them for the possibility of future success in professional and academic environments. Today, 14 years after its inception, Kidumatica numbers more than 500 members, 40% of them girls, from approximately 25 different homelands and 20 linguistic backgrounds, Jews and Arabs, new immigrants and native born, etc. Kidumatica's name has become a byword for mathematical excellence, as over the years its members have won multiple national and international Olympiads. Moreover, hundreds of Kidumatica graduates now populate the universities in mathematically oriented subjects, working towards the promise of the social and economic opportunity these fields can offer.

3. Three pillars of success: giftedness, creativity and multiculturalism

Everything in Kidumatica – from its initial identification of prospective students

to the contents and teaching methods of its day-to-day activities – revolves around the three pillars of giftedness, creativity and multiculturalism.

(a) Potential recruits for the club are identified using a specially designed method, tailor made to be fair and reliable, and to anticipate success.

(b) Learning is conducted in interactive workshops where students work in small groups.

(c) "Mini-courses" provide the students with "ingredients" for creativity, teaching them a series of mathematical skills and tools that can be drawn on and combined into creative solution strategies.

(d) The workshops rely on a "Problem Based Learning" (PBL) approach, a teaching method particularly suited to gifted students, which uses the solving of challenging, non-routine problems to foster students' creativity and actively generate new mathematical knowledge.

(e) They are taught by instructors who specialize in the specific topic that they teach (as opposed to the far more generalized specialization of school mathematics teachers), and who are also experts in working with gifted students (they know, for instance, what degree of frustration is conducive to learning, what to say explicitly and what to let the students grapple with alone, how to take higher mathematics and make them accessible to younger students, etc).

(f) The teaching staff is also supplemented with student mentors, university students whose job it is to serve as "big brothers" to the children, to serve as intermediaries between the students and their instructors and to provide them with the mathematical and emotional support that their parents are not always able to give them at home. We also make sure to have mentors who speak a variety of languages and help club members whose mother tongue is not Hebrew.

(g) Assessment in Kidumatica is not conducted in the form of grades, but rather as overall assessments that take into account additional factors like persistence, participation and contribution to group discussion.

4. **The rod vs. the fish – Problem Based Learning**
The following is a typical Kidumatica problem that illustrates the PBL approach and its particular suitability for encouraging creativity within a multicultural community of mathematically gifted students.
Paper Tearing Problem Doron takes a piece of paper and tears it into four pieces. Then, he takes one of the pieces and tears it into four pieces. Then he takes one of the pieces and tears it into four. And so Doron repeats the process again and again and again. By using this repeating process, will Doron eventually get 2012 pieces? If so, how many times will he have to repeat the process? If not, show why he cannot.
This problem is particularly suited to our goals in several ways. First, it is carefully designed to be culturally non-

specific. The tearing of paper is highly familiar and easy to visualize for members of all cultural group, even those who might not be familiar with problems based on things that seem everyday to us, like the slicing of pizza or the distribution of payments on a credit card. Secondly, the problem is specifically oriented towards mathematically gifted students in that it is based on the principle of "giving them the rod rather than the fish". Instead of providing our students with many instances of the same type of problem from which to deduce an automatic formula that may be used whenever another such problem may cross their path (in other words, with lots of fish), we at Kidumatica provide our students with a small number of explicit strategies or "rods", with which they may "fish" for the solution to any problem they encounter.

Kidumatica's approach is unique in that it provides its students with these basic solution strategies explicitly, openly citing both their names and how they may be used. Such strategic "tools" are usually not given to students by name so as not to burden them with excessive names, or fix their thinking in any one direction. Because our program is designed for mathematically gifted students, however, we aim to equip them with a set of tools, and to teach them the basic function of each. This method is key to fostering our students' creativity. Just as a carpenter must know the purpose and function of each of his tools before he can bend them to a variety of different tasks, so the Kidumatica students must learn the names and basic uses of the mathematical tools in their own toolbox before they can combine and adapt them creatively

to a variety of new situations. One of these tools is the "invariant strategy. The paper tearing problem is based on the invariant strategy. In the dynamic process described above, the key to comprehension lies in the stable, invariable element that stays the same through every repetition of the tearing action. This problem allows the students to look for the invariant in a visual or even tactile way, to physically tear a piece of paper several times, to translate the tearing into a table, and finally to deduce that each repetition adds three pieces to the total amount. This constant addition is the invariant, and from here the problem's solution is not far behind.

The paper tearing problem is fairly steeped in mathematical potential.

Firstly, comprehending requires the students to mathematize an action that is initially presented to them in a verbal and visual way. Second, to solve the problem they must understand and apply a useful and highly adaptable solution strategy finding the invariant. Third, having solved the problem, they must then defend their solution to their peers,developing and using a variety of argumentation and justification methods. Fourth, this problem constitutes a threshold from which to advance to a number of new mathematical topics, such as patterns, using verbal and symbolic formulas, moving at a later stage to the topic of arithmetic progression, and then further to modular systems and discrete math. Finally, this problem has enormous potential for creative generalization and the extrapolation of new problems from its basic design. The paper tearing problem's

potential as a basis for creating additional problems fuels the flame of the students' creative mathematical imaginations. The new problems it inspires take several forms. One of these is a simple expansion of the rules to a new invariant, such as tearing the paper into five pieces. Another is generalization which amounts of pieces do the problem's parameters allow? Which cannot be reached? Why? A third form is the creation of more complex combination rules, for instance: if we alternate between tearing the paper into three pieces one time and five pieces the next, can we reach 2012? Why or why not? In conclusion, problems like the Paper Tearing Problem exemplify the sum of Kidumatica's goals:

(a) The problem is completely non-reliant on culture or language.

(b) In both teaching method and content, it is tailored to mathematically gifted students, using engagement with the non-routine problem to teach an important solution strategy like the invariant.

(c) It is a basis from which to launch future mathematical learning, and for mathematical creativity at its best.

5. **Diamonds in the rough – Unbiased and accurate identification**

One of the challenges Kidumatica faces due the extensively multicultural community from which it draws its members is avoiding the cultural biases often found in standard entrance examinations. Kidumatica's application and identification process is therefore

specially designed for a maximum of fairness and equality ([5]).

To begin with, submission of potential applicants is based not only on the recommendations of teachers, but also on individual initiative. As is common practice for many programs for the gifted and talented, the club sends letters to all schools in the region, asking mathematics teachers to recommend students they think are suitable. While this method has the major advantage of reaching students who study in all schools in the region, it also has its disadvantages. For example, some teachers tend to view the more dominant pupils as talented and ignore the quiet ones. Also, bias might prevent teachers from recommending some of the students, especially members of minority groups ([4], [10]). To overcome this shortcoming, students are also invited to submit themselves, based on the assumption that students who are overlooked by their teachers but see themselves fit, and have the drive and motivation, deserve the same opportunity as their peers.

Kidumatica's entrance exams are also specifically designed to minimize cultural bias. First of all, the students are given more than one testing opportunity. Girls and minority students tend to experience math anxiety more than others, especially in testing situations, which can result in lower achievements ([7]). Therefore, the students are asked to participate in three different exams on two separate occasions. After facing the first exam session, the applicants are no longer "facing the unknown" and the anxiety level is reduced. Furthermore,

the exam tasks themselves do not rely exclusively on "school-math" (the level of which can vary extensively between schools of different socio-economic standing). Rather,the tasks are designed according to ([3]) to reveal those characteristics associated with mathematical talent – cognitive abilities such as data management, logical thought, pattern recognition, abstraction and reasoning, as well as other important traits such as persistence and motivation. The latter are important in assessing a student's intellectual stamina and will to succeed, even when faced with frustration and lack of knowledge. Tutors who speak the candidates' various mother tongues are present during the exams to overcome language barriers by explaining any words or terms that may be unknown.

Take, for example, this problem from the Kidumatica entrance exam:

Find the Pentagon: In Figure 1 there are two triangles.

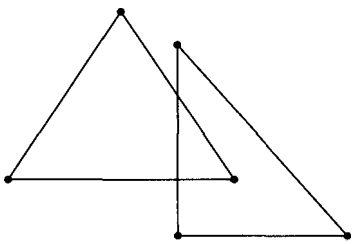

Figure 1: Find the Pentagon

The overlapping area is also a triangle. Draw two

triangles with an overlapping area that is a pentagon.

A problem like this one tests the students' visualization, as well as their persistence in the face of a challenge. Taking into account that not all of the students know what a pentagon is, or are familiar with the term "overlapping," terms such as these are freely explained to any student who requires this. Explaining these concepts to the students does not reduce the cognitive level of the task, and enables the examiners to look into the logical competence of the students with a minimal "interference" from cultural or linguistic obstacles.

Here is another problem for younger students, designed to assess certain qualities indicative of mathematical giftedness:

The sum of two (natural) numbers is odd. Their product will be:

(a) always odd

(b) always even

(c) sometimes even and sometimes odd.

Justify your answer.

Solving a problem like this indicates that the student has a meaningful understanding of the term "odd number," and of what multiplication is. It also requires an intuitive and intelligent use of the distributive and associative laws, as well as creativity in posing arguments to support their conclusion. Different approaches to this problem range from a simple list of numeric examples to verbal generalization and creative imagery. For example, one student explained: "an even number can be split into

two equal parts. If we multiply each part separately you get the same thing. Then you add the two equal products together. If you add together two equal numbers you get something even."

6. **Placing process over end result – Fairness and equality in exam grading**

   The grading of the exams and the final selection of applicants is conducted according to a combination of absolute and relative parameters. While correct answers to the exam questions are allotted maximum points, credit is also given to applicants based on their solution paths and justifications, since we assume that if we were to look only for correct answers, crucial information about students' cognitive abilities might be overlooked ([5]).

   Acceptance to Kidumatica is absolute to the extent that the students who achieve the highest scores overall are accepted. However, special care is also taken to make sure that some students from every school that recommended students will be accepted. Thus, the students with the highest grade from each school are also accepted, even if they did not score high overall, since they are best in relation to their environment and the standards of their school. This provides all of the students with an equal opportunity of acceptance, regardless of their mathematical background.

7. **Kidumatica – Cultivating a legacy**

   In the years since its inception, Kidumatica has become a laboratory for the cultivation of creativity and multiculturalism. Moreover, its reach has spread beyond

education into the realm of research, with the club also serving as fertile ground for numerous PhD studies, researching new teaching methods for gifted students, and gauging their ability to engage with topics such as critical thinking, probability, modeling and more. This continual cycle of research, assessment and improvement has produced teaching programs with the proven power to foster giftedness and creativity in Kidumatica's multi-cultural community.

The efficacy of the Kidumatica method is evident in comparative studies conducted between club members and non-members with similar potential. While these studies showed no difference in the solution of school-type problems, the Kidumatica students has an advantage when facing challenging and non-routine problems.

The Kidumatica model and the knowledge it has generated are already being disseminated, in part or in full, in various places around the world. Identifying gifted students fairly, without any social or cultural bias, is an acute challenge faced by many institutions a challenge to which Kidumatica has successfully risen. For the various inhabitants of this relatively poor, provincial and arid southern region, opportunity can often be as scarce as water. But as the many Kidumatica graduates now populating the country's universities have shown, Kidumatica's methods of finding and promoting creativity and excellence can break through the social and cultural barriers standing in these children's way and help bring their potential to its fullest flower.

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RIGOROUS ANALYSIS OF BRAHMAGUPTA'S GEOMETRY IN BHĀSKARĀCĀRYA'S LĪLĀVATĪ[1]

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ABSTRACT

Bhāskara II (1150), the influential mathematician of 12th century, considered Brahamagupta's result for the area of a cyclic quadrilateral (*tricaturbhuja*) as inexact and his results on circumradius (*hṛdayaraǰju*), [Br.Sp.Si.XII.26-27], the radius of the circumcircle, were even omitted. As a result, the nature of his "triquadrilateral" (*tricaturbhuja*) (i.e.cyclic quadrilateral) was ignored.

Though this was regrettable in itself, it had a salutary effect also. In that it led to a consideration of the general quadrilateral, which had been hither to left untouched and also prompted the idea of third diagonal in a cyclic quadrilateral.

The geometry of three diagonals of a cyclic quadrilateral is discussed in greater detail by Nārāyana Pandita. The mathematicians of the Kerala-Āryabhata school clearly understood the scope and applicability of Brahmagupta's results.

I. BRAHMAGUPTA :

Brahmagupta holds a unique position in the history of Ancient Indian Mathematics. He was born in a village called Bhillamala in North West Rajasthan in the year 598 AD and wrote his first book *Brāhmasphuṭasiddhānta (Br.Sp.Si.)* in the year 628 AD. He wrote a second book *Khaṇḍakhādyaka (Kh.Kh.)* later in 665 AD. The first book contains 1008 *ślokas* (verses) in 25 chapters. The second book has 194 *ślokas* and deals with astronomical calculations in 9 chapters.

1.1 : *Brāhmasphuṭasiddhānta*

The purpose of *Brāhmasphuṭasiddhānta (Br.Sp.Si)* was to establish a corrected form of an older astronomical system, and to offer a rebuttal of other systems current in India at that time, including that of Āryabhata I (499 AD). The mathematical portions of *Br.Sp.Si.* (including Chapters XII, XVIII, XIX and XXI) document the conceptual advances that Brahmagupta or his predecessors had achieved. Brahmagupta has called the twelfth chapter of *Brāhmasphuṭasiddhānta* as *Gaṇita* and the eighteenth chapter as *Kuṭṭaka*.

According to tradition of those times, *Gaṇita* includes matter pertaining to arithmetic problems on mixtures, plane figures, shadows, series, piles and excavations. Pṛthūdakasvāmy (860 A.D.) proves some of the rules of *Brāhmasphuṭasiddhānta* and also gives examples to the rules of *Gaṇitādhyāya*.

The Eighteenth Chapter of *Brāhmasphuṭasiddhānta*, the *Kuṭṭakādhyāya*, contains solutions of the indeterminate equations of both first and second degree.

---

[1] Prof R.C.Gupta Endowment Lecture,
50th Annual Conference, Association of Mathematics Teachers of India (AMTI)

The *Spaṣṭādhikāra* chapter contains trigonometrical notations including standard tables of sines and versed sines.

1.2 : *Khaṇḍakhādyaka*

Brahmagupta was the first to give second order interpolation formulas for equal as well as un equal tabulated argumental intervals. Mathematically, his rule is equivalent to the modern Newton–Stirling interpolation formula up to second order. Thus a new branch of mathematics – "Interpolation theory" was initiated by Brahmagupta.

1.3 : *Gaṇitādhyāya* of *Brāhmasphuṭasiddhānta* :

He contributed such elegant results to Geometry and Number Theory that today's mathematicians still marvel at their originality. His theorems leading to the calculation of area, and the lengths of the diagonals of a cyclic quadrilateral, construction of a rational cyclic quadrilateral are certainly the hallmarks of a genius.

Bhāskarācārya described Brahmagupta as *"Gaṇaka cakra cūḍāmaṇi"* (Jewel among the circle of mathematicians).[1]

II. BRAHMAGUPTA'S RESULTS :

The most important geometrical contribution of Brahmagupta are related to cyclic quadrilaterals.[2]

2.1 : Area Theorem: He was the first in the world to give the exact formula for the area of such figures in terms of sides. He says very concisely:

स्थूलफलं त्रिचतुर्भुज बाहुप्रतिबाहुयोगदलघातः ।
भुजयोगार्धचतुष्टय भुजोनघातात्पदं सूक्ष्मम् ॥२१॥

sthūlaphalaṁ tricaturbhuja bāhupratibāhuyōgadalaghātaḥ |
bhujayōgārdhacatuṣṭaya bhujōnaghātātpadaṁ sūkṣmam ||21||

"The product of half the sum of (the two pairs of) the opposite sides of a *tricaturbhuja* gives the gross area. Set down half the sum of the sides in four places (and) diminish them by the four sides (respectively). The square root (of the product of the four remainders) is the accurate area."

That is, If a, b, c, d are the sides of a cyclic quadrilateral then,

$$\text{The gross area} = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right) \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\text{The accurate area} = \sqrt{(s-a)(s-b)(s-c)(s-d)} \quad \ldots\ldots\ldots \quad (2)$$

$$\text{where } s = (a+b+c+d)/2 \quad (3)$$

C.B. Boyer considers this formula (2) to be "perhaps most beautiful result". But equally beautiful are :

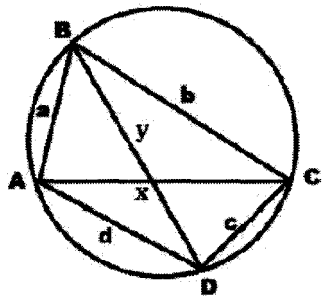
2.2 : Brahmagupta's expression for the diagonals:

कर्णाश्रितभुजघातैक्यमुभयथाऽन्योन्य भाजितं गुणयेत् ।
योगेन भुजप्रतिभुजवधयोः कर्णौ पदे **विषमे** ॥२८॥

karṇāśritabhujaghātaikyamubhayathā'nyonya bhājitaṁ guṇayēt |
yōgēna bhujapratibhujavadhayōḥ karṇau padē viṣamē ||28||

" Divide mutually the sums of the products of the sides attached to both the diagonals and then multiply the quotients by the sum of the products of the opposite sides : the square roots of the results are the diagonals in an asymmetric (*viṣama*) quadrilateral."

In a cyclic quadrilateral ABCD, if a, b, c, d are the lengths of the sides AB, BC, CD and DA respectively, and its diagonal AC = x and BD = y, then according to Brahamagupta's theorem.



$$AC = x = \sqrt{\frac{(ad+bc)}{(ab+cd)}(ac+bd)} \quad \ldots \ldots (4)$$

$$BD = y = \sqrt{\frac{(ab+cd)}{(ad+bc)}(ac+bd)} \quad \ldots \ldots (5)$$

Thus enabling the diagonals to be calculated in terms of the sides.

Brahmagupta's these expressions for the diagonals are considered to be the "most remarkable in Hindu geometry and solitary in its excellence" by a recent historian of mathematics. The second among the two formulas given above was rediscovered in Europe only a thousand years later by W. Snell (about 1619 AD).[3]

2.3 : Brahamaguptan Quadrilaterals :

A note worthy contribution of Brahmagupta to pure mathematics is his method of forming what are called the Brahmaguptan Quadrilaterals. These are cyclic quadrilaterals in which the sides, diagonals and area are all rational (or integral) and the diagonals intersect orthogonally.

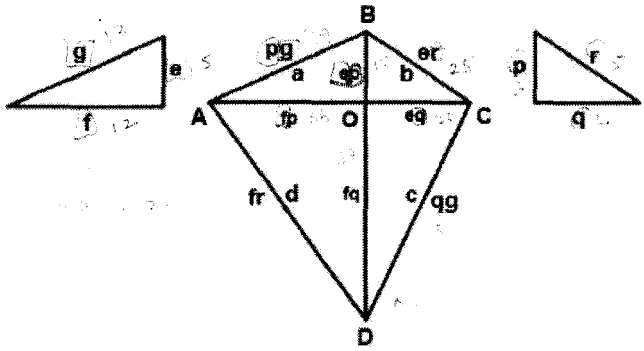
जात्यद्वयकोटिभुजाः पर कर्ण गुणा भुजश्चतुर्विषमे ।
अधिको भूर्मुखहीनौ बाहुद्वितयं भुजावन्यौ ॥३८॥

jātyadvayakōṭibhujāḥ para karṇa guṇā bhujaścaturviṣamē |
adhikō bhūrmukhahīnau bāhudvitayaṁ bhujāvanyau ||38||

"The kotis (uprights) and bhujas (bases) of two jātyas (rational right triangles) multiplied by each other's hypotenuse are the four sides in a viṣama (un-equal) quadrilateral (in which the diagonals intersect orthogonally). The longest is the base, the least is the face and the *remaining two sides, the flanks.*"

If e, f, g and p, q, r are the sides (integral or rational) of two (jātyas i. e.) right triangles ('g' and 'r' being hypotenuses), then er, fr, gp, and gq are the required sides of a Brahmaguptan quadrilateral. The figure was intelligently constructed from the four triangles with sides : (pe, pf, pg) ; (ep, eq, er) ; (qe, qf, qg) and (fp, fq, fr) as shown in fig.

Actually, any quadrilateral whose sides are *gp , er , gq ,* and *fr* taken in any order is cyclic. Its diagonals cut at right angles if the greatest and least sides are placed opposite to each other. Moreover, if the diagonals do not cut at right angles, one of them will be *'gr'* and will pass through the centre of the escribed circle. Then the greatest and least sides will be adjacent sides, instead of opposite sides.



Taking (3 , 4 , 5) and (5 , 12 , 13) for *(p , q , r)* and *(e , f , g)* we obtain the Brahmaguptan quadrilaterals with sides (25 , 39 , 60 , 52) and (25 , 60 , 39 , 52). This example is mentioned in Srīdhara's (750 A.D.)*Triśatika* (Ex. 80). The same example is given by Mahāvīra (850. A.D.) (G.S. S.VII-104½), Pṛthūdakasvāmi and Bhāskara II. This particular Brahmagupta quadrilateral went to the west in a slightly transformed form and was the only rational cyclic quadrilateral known in the west till the time of Euler (c. 1750. A.D.)[4]

An anonymous commentator of Nārāyana's *Tantrasāra* gives a clue for the correct appreciation of Brahmagupta's rule. According to him one method of forming Brahmagupta's rational cyclic quadrilateral is to juxtapose two rational right triangles having the same hypotenuse, with their hypotenuse coinciding.[5] For example take two different rational right angled triangles, (say 3 , 4 , 5 and 5 , 12 , 13) and from them two other right triangles are obtained by multiplying the sides of one triangle by the other's hypotenuse. The new right triangles will be (39 , 52 , 65) ; (25 , 60 , 65) and they have the same hypotenuse and can therefore be juxtaposed with their hypotenuses coinciding.

Then that hypotenuse will be a diagonal of the quadrilateral as also a diameter of the escribed circle. By interchanging the sides which are chords of circumscribing circle other cyclic quadrilaterals are obtained. But, in all these, either the diagonals will cut at right angles or else one of the diagonals will be a diameter of the escribed circle,

Moreover, in these quadrilaterals *'the sum of the squares of the largest and least sides will be equal to the sum of the squares of the remaining sides.'*[6]

By taking the right triangles (3 , 4 , 5) and (15 , 8 , 17) Bhāskara-II gets another cyclic quadrilateral (68 , 51 , 40 , 75) , whose diagonals are (77 , 85) and area is 3234. With the sequence of the sides (68 , 40 , 51 , 75) The diagonals are (77 , 84); while with (68 , 40 , 75 , 51) they are (84 , 85). The deep significance of Brahmagupta's results has been demonstrated by chasles and Kummer.[7]

III. BHĀSKARĀCĀRYA[8]:

The greatest name in the history of ancient and medieval Indian astronomy and mathematics is that of Bhāskarācārya (born 1114 A.D.). His *Līlāvatī* is the most popular book of traditional Indian mathematics. He is usually designated as Bhāskarā II in order to differentiate him from his earlier name sake the Āryabhatan scholiast Bhāskara I who wrote a commentary in A.D. 629 on the *Āryabhaṭīya* of Āryabhaṭa I (born A.D.476)

According to Bhāskarācārya's own statement towards the end of his *Gōlādhyāya*, he was born in śaka 1036 or (A.D.1114); and composed the work *Siddhānta- Śiromaṇi* (with his own exposition called *Vāsanabhāṣya* or *Mitākṣara*) at the age of 36 years (i.e. in A.D. 1150). His father Maheśvara was also his teacher as well. He also adds that he came from Vijjaḍaviḍa near the Sahya mountain. Usually this Vijjaḍaviḍa is identified with modern Vijayapur (or Bijapur) in Karnataka. Abual Fayd-Fayadi, the translator of *Līlāvatī*, in his Persian translation work stated that, Bhāskarācārya hailed from Vedar (present Bidar belonging to Karnataka).

Usually it is customary to regard *Līlāvatī*, *Bījagaṇita*, *Grahagaṇita* and *Gōlādhyāya* as the four parts of the *Siddhānta-Śiromaṇi* to make it a comprehensive treatise of Bhāskarācārya's Hindu mathematical sciences. Bhāskarācārya grand son Caṅgadeva (who was the chief astronomer in the court or the King Singhaṇa) had established in śaka 1129 (or A.D. 1207), a maṭh (residential institution) for the study of the works of his grand father.

LĪLĀVATĪ

3.1 : *Līlāvatī* (The Beautiful):

The popularity of *Līlāvatī* is shown by the fact that it is still used as a text book in the Sanskrit medium Institutions throughout India. Ever since its composition, the *Līlāvatī* has been commented by a large number of scholars. Some of the Sanskrit commentators were: Gaṅgādhara (ca.1420), Parameśvara (ca.1430), Lakṣmīdāsa(ca.1500), Sūryadāsa(ca.1540), Gaṇeśa (1545), Mahīdhara (ca.1590), Munīśvara (ca.1635), Rāmakṛṣṇa (1687), etc. The Kriyākramakarī (ca.1534) commentary is a joint work of Śaṅkara Vāriyar and Mahiṣamaṅgala Nārāyaṇa (who completed it after the demise of Śaṅkara). This commentary is a rich source for traditional Indian mathematics.

3.2 : Kṣetra-vyavahāra :

In *Līlāvatī*, Kṣetra-vyavahāra, the section on closed figures begins with verse 135, and 135-168 deal with trilaterals and that from 169-190 with quadrilaterals; while 191-198 with Sūcīkṣetras, and finally 199-213 with circle and sphere.

3.2.1 : Trilateral or Triangle :

After defining bhuja and koṭi in a trilateral and a quadrilateral [vs.135], Bhāskara II states the The Bhuja-Koṭi-Karṇa Nyāya or the diagonal-square-theorem (or the so called Pythagoras theorem) in its three different forms[136]. To ease the calculations he introduces two algebraic identities viz: $(a-b)^2 + 2ab = a^2 + b^2$ and $(a+b)(a-b) = a^2 - b^2$ [138], and further gives a method to find the approximate value of square-root of a non-square number [140]. He shows how from any two sides of a right triangle the third can be

calculated [141-142]. In his treatment of rational figures, he has a new solution for the sides of a right triangle with one side containing the right angle given [141]. The integral solutions of right triangle given by Bhāskara II using two arbitrary quantities, bījas [147] ,is the same as that given by Brahmagupta [Br.Sp.Si. XII 33].

Bhāskara's enunciation and elaboration of the theorem of the square of the hypotenuse is intended to make it useful in solving everyday problems such as :

i) The Broken Bamboo Problem (Vaṃśabhaṅgoddeśaka) [149]
ii) Peacock And Serpent (Sarpa-Śikhaṇḍi) or Hawk And Rat (Śyena- Mūṣaka) problem [151]
iii) Sinking Of Lotus (Kamaloddeśaka) [154-155]
iv) Vertical-Leap (Uḍḍīyamāna) problem [156]
Further, in a triangle hypotenuse being known, as also the sum of the side and upright or their difference, he has given rules to discriminate those sides [158].

Bhāskara II after stating the condition for the formation of a closed rectilinear figure [163], gives formulae for calculating the ābādhās, (the segments of the base made by the altitude or rather the projections of the sides on the base) and the altitude[165-166]. He also gives an example involving negative projection (ṇa-ābādhā)[168]. He has nothing new to add on triangle. His achievements were in the circle and sphere.[9]

3.2.2 : General Quadrilateral :

Bhāskara II has given general formula for the exact area of a square, rectangle, rhombus and trapezium [173]. But he restricts Āryabhata(II)'s expression for the diagonals of a parallelogram in terms of the other diagonal and the sides to the quadrilateral with all sides equal, i.e. to the square and the rhombus [172].

For calculating the diagonals in a quadrilateral, the altitudes at the ends of the diagonals are used, when the calculation reduces to a straight forward application of the theorem of the square of the diagonal [179].

Bhāskara II not only gives a rule for finding the second diagonal when the first is given [180-181], in addition he investigates the condition that has to be satisfied by the diagonals of a quadrilateral i.e. the upper and lower limits of their length [182].

He gives a rule for the exact area of a general quadrilateral wherein, the areas of triangles into which a diagonal cuts a quadrilateral are to be computed separately and added [183]. Bhāskara II adopts Śrīdhara's method [*Triśatikā*. R.49][10] of calculating the altitude of a trapezium from the triangle formed with the difference between parallel sides as base and the flanks of the trapezium as sides, for finding its altitude and ābādhās[184-185(i)]. He further investigates the possibility of any four lengths forming a trapezium and gives the condition that : *In a trapezium the sum of the other side and face is smaller than the sum of the smaller flank-side and the base*[185(ii)].

3.2.3 : Cyclic-Quadrilaterals

Observations made and the remarks passed by Bhāskara II on Brahmagupta's expressions for the area [169] and the measure of diagonals of a cyclic-quadrilaterals [190] show that he failed to realize the true significance of them. However it resulted in the study of general quadrilateral and prompted the idea of third diagonal [gloss on 190] in a cyclic quadrilateral. [See. Section VI below]

IV. BHĀSKARĀCĀRYA'S FORMULAS

4.1: Formulas for ābādhās, and avalamba in a triangle :

Bhāskara II repeats the formulae, stated by the earlier scholars, for calculating the segments of the base (ābādhās), made by the altitude (avalamba) and the altitude[11], and uses it extensively. It is as follows :

त्रिभुजे भुजयोर्योगस्तदन्तरगुणो भुवा हृतो लब्ध्या ।
द्विःस्था भूरूनयुता दलिताऽऽबाधे तयोः स्यातम् ॥१६५॥
स्वाबाधाभुजकृत्योरन्तरमूलं प्रजायते लम्बः ।

tribhujē bhujayōryōgastadantaraguṇō bhuvā hṛtō labdhyā |
dviḥsthā bhūrūnayutā dalitāऽऽbādhē tayōḥ syātām ||165||
svābādhābhujakṛtyōrantaramūlaṁ prajāyatē lambaḥ |

"In a triangle, the sum of two sides being multiplied by their difference, is divided by the base ; the quotient is subtracted from, and added to, the base which is twice set down : and being halved, the results are segments corresponding to those sides. The square root of the difference of the squares of the side and its own segment of the base becomes the perpendicular."

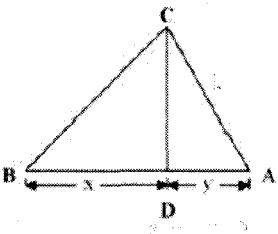
In triangle ABC, CD is the the altitude and BD and AD are the ābādhās.
Let $BC = a$, $AC = b$, $BA = c$, $CD = p$, $BD = x$, *and* $AD = y$.
Then as per the rule:

Step I. : $\frac{(a+b)(a-b)}{c}$

Step II. $\frac{c-\frac{(a+b)(a-b)}{c}}{2}$ $\qquad$ $\frac{c+\frac{(a+b)(a-b)}{c}}{2}$

Step III. $y=\left(\frac{1}{2}\right)\left\{c-\left(\frac{a^2-b^2}{c}\right)\right\}$; $x=\left(\frac{1}{2}\right)\left\{c+\left(\frac{a^2-b^2}{c}\right)\right\}$

Step IV. $CD = p = \sqrt{b^2-y^2} = \sqrt{a^2-x^2}$



4.2 : Formula for the Second diagonal :

Bhāskara II gives an expression for finding the second diagonal when the first is known or given[12] :

इष्टोऽत्र कर्णः प्रथमं प्रकल्प्यस्त्र्यस्रे तु कर्णोभयतः स्थिते ये ।
कर्णौ तयोः क्ष्मामितरौ च बाहू प्रकल्प्य लम्बावबधे च साध्ये ॥१८०॥
आबाधयोरेककुपस्थयोर्यत् स्यादन्तरं तत्कृतिसंयुतस्य ।
लम्बैक्यवर्गस्य पदं द्वितीयः कर्णो भवेत्सर्वचतुर्भुजेषु ॥१८१॥

iṣṭōऽtra karṇaḥ prathamaṁ prakalpyastryasrē tu karṇōbhayataḥ sthitē yē |
karṇaṁ tayōḥ kṣmāmitarau ca bāhū prakalpya lambāvabadhē ca sādhyē ||180||
ābādhayōrēkakakupsthayōryat syādantaraṁ tatkṛtisaṁyutasya |
lambaikyavargasya padaṁ dvitīyaḥ karṇō bhavētsarvacaturbhujēṣu ||181||

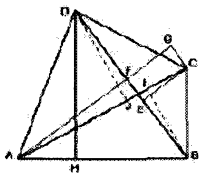
"In a quadrilateral, assume the value of one diagonal. Then in the two triangles lying on either sides of this diagonal, it will be the base and others (i.e. the given sides of the quadrilateral) sides. Now find the perpendiculars and segments (in these triangles) . Then the

square of the difference of the two segments lying on the same side (i.e. taken from the same corner) being added to the square of the sum of the perpendiculars, the square- root of the resulting sum will be the second diagonal in all quadrilaterals."

Let $ABCD$ be a quadrilateral whose diagonal BD as well as the sides are known.

Draw AF, CE perpendicular to BD.

Produce AF and draw CG perpendicular to it. Join AC.



Then, $AC^2 = AG^2 + CG^2$

$= (AF + CE)^2 + (BF - BE)^2$

4.3 : Upper and lower limits for arbitrary assumption of a diagonal :

Bhāskara II investigates the condition that has to be satisfied by the diagonals of a quadrilateral.

कर्णाश्रितस्वल्पभुजैक्यमुर्वी प्रकल्प्य तच्छेषभुजौ च बाहू ।
साध्योऽवलंबोऽथ तथान्यकर्णः स्वोर्व्याः कथंचिच्छ्रवणो न दीर्घः ॥
तदन्यलंबान्न [/तदन्यकर्णान्न] लघुस्तथेदं ज्ञात्वेष्टकर्णः सुधिया प्रकल्प्यः ॥१८२॥

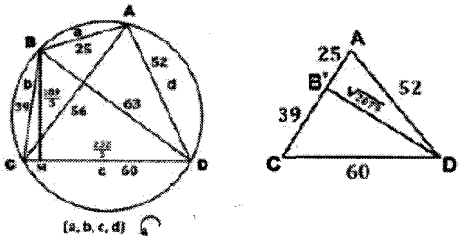
karṇāśritasvalpabhujaikyamurvīṁ prakalpya tacchēṣabhujau ca bāhū |
sādhyōऽvalambōऽtha tathānyakarṇaḥ svōrvyāḥ kathaṁcicchravaṇō na dīrghaḥ ||
tadanyalambānna [/tadanyakarṇānna] laghustathēdaṁ jñātvēṣṭakarṇaḥ sudhiyā prakalpyaḥ ||

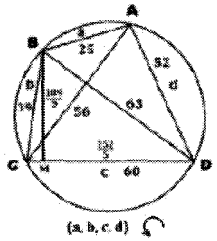
"With the smaller of the sums of the sides about the diagonal as base and other sides as sides the altitude is to be calculated, similarly the other diagonal. The other diagonal can not in any case be longer than its base (i.e. the base of the triangle formed as above), nor can the first be shorter than its altitude / diagonal. Adverting to these limits , an intelligent person may assume a diagonal."

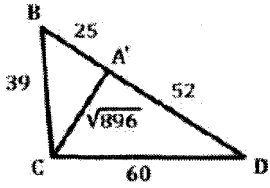
The process of arriving at these conditions is explained by Bhāskara himself :

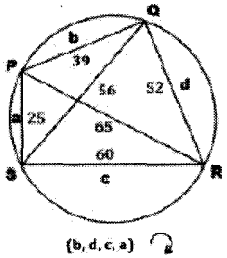
चतुर्भुजे ह्येकान्तरकोणावाक्रम्य संकोच्यमानं त्रिभुजत्वं याति । तत्रैककोणे लग्नलघुभुजयोः ऐक्यं भूमिम् इतरौ भुजौ च प्रकल्प्य तल्लंबादूनः संकोच्यमानः कर्णः कथंचिदपि न स्यात्ततोऽसौ भूमेरधिको न स्यादेवमुभयथाप्येतदनुक्तमपि बुद्धिमता ज्ञायते । 182 com.

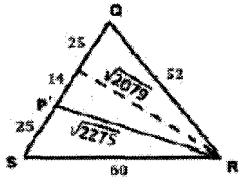
"For a quadrilateral, contracting as the opposite angles (corners) approach, becomes a triangle; wherein the sum of the least pair of sides about one angle is the base, and the other two are taken as the sides. The perpendicular is found in the manner before taught. Hence the shrinking diagonal cannot by any means be less than the perpendicular ; nor the other greater than the base. It is so both ways. This, even though it were not mentioned, would be readily perceived by the intelligent student."



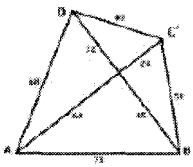


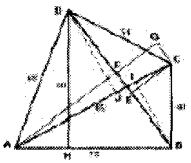


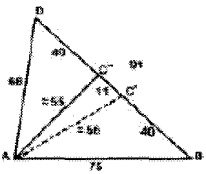




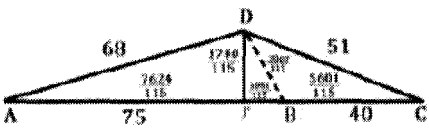
$\sqrt{\mathbf{896}} \approx \mathbf{30} < AC < 64 (= 39 + 25)$ $\sqrt{\mathbf{2079}} \approx \mathbf{45}\frac{\mathbf{3}}{\mathbf{5}} < BD < 77 (= 52 + 25)$







$\sqrt{\mathbf{3146}} \approx \mathbf{56} < AC < 115 (= 75 + 40)$ $\frac{\mathbf{2007}}{\mathbf{115}} = \mathbf{17}\frac{\mathbf{52}}{\mathbf{115}} < BD < 91 (= 51 + 40)$



In this connection, it is mentioned in the foot notes of English translation (Ed. by HCB.p.119) that :

"BD cannot be longer than DC + BC, but always shorter ; **nor it can be shorter than the perpendicular DH**, but always longer". The second half of the remark is not correct.

V. The Study Of Quadrilaterals : (Cyclic and non cyclic) in India :

5.1 : Two sections of Indian mathematician have approached the study of the quadrilateral from two different angles. One section viewed it merely as a figure enclosed by four chords of a circle, whereas the other viewed it as a figure enclosed by any four lines, i.e. The general quadrilateral but strangely enough excluding the cyclic quadrilateral. The former includes Brahmaguta (628), Śrīdhara (c.750), Mahāvīra (850), Śrīpati (1039) and Kerala- Āryabhaṭīya school ; the latter Āryabhaṭa II (c.950) and Bhāskara II (1150). Nārāyaṇa Paṇḍita (1356) has leaning towards both.[13]

5.2 : Unfortunately Brahmagupta's results were misunderstood by his immediate successors. His result for the area of a Cyclic quadrilateral (*tricaturbhuja*) was considered to be inexact in the 12th century by the influential mathematician Bhāskara II. Brahmagupta's results on the radius of the circum-circle, circum-radius (*hṛdayarajju*) [Br.Sp.Si.XII.26-27)], were even omitted; as a result, the nature of his "triquadrilateral" *(tricaturbhuja)* (i.e. Cyclic quadrilateral) was ignored.

Perhaps due to lack of coordination of mathematical knowledge in the different schools, the true significance of the theorem failed to be grasped by Bhāskara II and Āryabhaṭa II . Though this was regrettable in itself, it had a salutary effect also, in that it led to a consideration of the general quadrilateral, which had been hither to left untouched.[14] **This also prompted the idea of third diagonal in a cyclic quadrilateral.**

VI. BHĀSKARĀCĀRYA'S CRITICISM ON BRAHMAGUPTA'S RESULTS :

6.1 : Bhāskara-II reproduces Brahmagupta's theorem for the area of a (cyclic) quadrilateral[15] and [considering the import of the technical term *'tricaturbhuja'* as a 'trilateral and (unrelated) quadrilateral'] adds that: the area so obtained will be gross. He states :

सर्वदोर्युतिदलं चतुःस्थितं बाहुभिर्विरहितं च तद्घात् ।
मूलम्-अस्फुटफलं चतुर्भुजे स्पष्टमेवमुदितं त्रिबाहुके ॥१६९॥

sarvadōryutidalaṁ catuḥsthitaṁ bāhubhirvirahitaṁ ca tadvadhāt |
mūlamasphuṭaphalaṁ caturbhujē spaṣṭamēvamuditaṁ tribāhukē ||169||

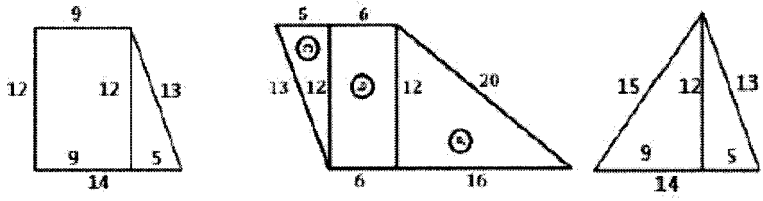
"Half the sum of all the sides is set down in four places; and the sides are severally subtracted. The remainders being multiplied together, the square root of the product is the area inexact in the quadrilateral, but pronounced exact in the triangle".

In other words, the area of a quadrilateral obtained by the above formulae (2) and (3) is gross, while with $d = 0$ in (2) and (3), will yield the exact area of a triangle. That is , in case of a triangle :

$$\text{The accurate area} = \sqrt{(s-a)(s-b)(s-c)(s)} \quad \ldots \qquad (6)$$

$$\text{where } s = (a+b+c)/2 \quad \ldots \qquad (7)$$

To corroborate his statements, he calculates the actual areas as : 138 and 198; and the areas according to the above formulae as : 141 and 250 , of the two trapeziums (9, 12, 14, 13) ; (11, 13, 22, 20) with parallel sides as (9, 14); and (11, 22) respectively. Also, the accurate area of the triangle (15, 14, 13) with 14 as the base is : 84.



6.2. : Bhāskara-II has taken much pains to show that the diagonal of a quadrilateral whose four sides only are given are not determinate.

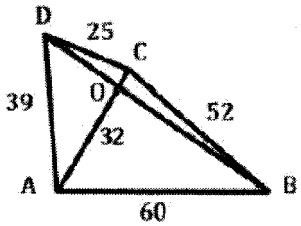
6.2.1 : Considering a quadrilateral formed by the four sides 25, 39, 60, 52 (having lengths equal to that of the sides of Brahmaguptan quadrilateral) [Ex. vs.186-187] and in which taking value of one of the diagonals as 32, works out the other diagonal by the rule stated by him [in vs.180-181] for finding the second diagonal of a quadrilateral.

$$AO \text{ or } CO = \frac{1}{2}\left\{32 \pm \frac{39^2 - 25^2}{32}\right\} = \frac{1}{2}\left\{32 \pm \frac{1521 - 625}{32}\right\} = \frac{1}{2}\left\{32 \pm \frac{896}{32}\right\}$$
$$\therefore AO = 30 \text{ and } CO = 2$$

Again, $AO \text{ or } CO = \frac{1}{2}\left\{32 \pm \frac{60^2 - 52^2}{32}\right\} = \frac{1}{2}\left\{32 \pm \frac{3600 - 2704}{32}\right\} = \frac{1}{2}\left\{32 \pm \frac{896}{32}\right\}$
$$\therefore AO = 30 \text{ and } CO = 2$$

Since the projections of BA and DA, and that of BC and DC are equal to AO and CO respectively , also BO and DO are perpendiculars drawn on AC, the points B, O, D are co-linear and hence the point O lies on the diagonal BD.

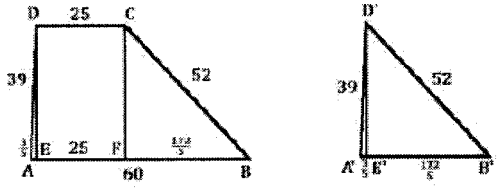
$$\therefore BD = BO + DO$$
$$= \sqrt{60^2 - 30^2} + \sqrt{39^2 - 30^2}$$
$$= \sqrt{3600 - 900} + \sqrt{1521 - 900}$$
$$= \sqrt{2700} + \sqrt{621} = \sqrt{\frac{2700\ 00\ 00}{100\ 00}} + \sqrt{\frac{621\ 00\ 00}{100\ 00}}$$
$$= \frac{5196}{100} + \frac{2492}{100} \quad = 51\frac{24}{25} + 24\frac{23}{25} = 76\frac{22}{25}$$



$$\therefore AC = 32 \text{ and } BD = 76\frac{22}{25}$$

It is interesting to observe here that the diagonals are mutually perpendicular in this case also.

6.2.2 : Again, considering the same four sides, viz, 25, 39, 60, 52 forms a trapezium with the sides 25 and 60 as parallel sides, first he works out its altitude by Sridhara's method [*Triśatikā*. R.49] and then its two diagonals.



$$A'E' \text{ or } B'E' = \frac{1}{2}\left\{35 \pm \frac{52^2-39^2}{35}\right\} = \frac{1}{2}\left\{\frac{1225\pm(2704-1521)}{35}\right\} = \frac{1}{2}\left\{\frac{1225\pm1183}{35}\right\}$$

$$\therefore A'E' = \frac{1}{2}\left\{\frac{1225-1183}{35}\right\} = \frac{1}{2}\left(\frac{42}{35}\right) = \frac{3}{5} \text{ and } B'E' = \frac{1}{2}\left\{\frac{1225+1183}{35}\right\} = \frac{1}{2}\left(\frac{2408}{35}\right) = \frac{1204}{35} = \frac{172}{5}$$

$$DE = \sqrt{39^2 - \left(\frac{3}{5}\right)^2} = \sqrt{1521 - \frac{9}{25}} = \sqrt{\frac{38025-9}{25}} = \sqrt{\frac{38016}{25}}$$

$$= \frac{1}{5}\sqrt{\frac{38016\,00\,00\,00}{1\,00\,00\,00}} = \frac{194976}{5000} = 38\frac{4976}{5000} = 38\frac{622}{625}$$

$$EB = AB - AE = 60 - \frac{3}{5} = \frac{297}{5} \text{ and } FA = AB - BF = 60 - \frac{172}{5} = \frac{128}{5}$$

$$BD = \sqrt{EB^2 + DE^2} = \sqrt{\left(\frac{297}{5}\right)^2 + \left(\sqrt{\frac{38016}{25}}\right)^2} = \sqrt{\left\{\frac{88209}{25} + \frac{38016}{25}\right\}}$$

$$= \sqrt{\frac{126225}{25}} = \sqrt{5049} = \sqrt{\frac{504900\,00}{100\,00}} \approx \frac{7105}{100} = 71\frac{5}{100} = 71\frac{1}{20}$$

$$AC = \sqrt{FA^2 + CF^2} = \sqrt{\left(\frac{128}{5}\right)^2 + \left(\sqrt{\frac{38016}{25}}\right)^2} = \sqrt{\frac{16384}{25} + \frac{38016}{25}}$$

$$= \sqrt{2176} = \sqrt{\frac{2176\,00\,00}{100}} \approx \frac{4665}{100} = 46\frac{65}{100} = 46\frac{13}{20}$$

Thus, $BD = 71\frac{1}{20}$ and $AC = 46\frac{13}{20}$

Thus obtaining Two sets of diagonals for the same four sides he remarks :

6.3 : Bhāskara-II remarks[16] :

चतुर्भुजस्यानियतौ हि कर्णौ कथं ततोऽस्मिन्नियतं फलं स्यात् ।
प्रसाधितौ तच्छ्रवणौ यदाद्यैः स्वकल्पितौ तावितरत्र न स्तः ॥
तेष्वेव बाहुष्वपरौ च कर्णावनेकधा क्षेत्रफलं ततश्च ॥१७१॥

caturbhujasyāniyatau hi karṇau kathaṁ tatō'sminniyataṁ phalaṁ syāt |
prasādhitau tacchravaṇau yadādyaiḥ svakalpitau tāvitaratra na staḥ ||
tēṣvēva bāhuṣvaparau ca karṇāvanēkadhā kṣētraphalaṁ tataśca ||171||

"The diagonals of a quadrilateral (whose four sides are given) are uncertain. How can, then, the area be determinate ? The diagonals as calculated by previous teachers will be true only in case of quadrilaterals (of a particular kind) contemplated by them, but not incase of others. For With the same (four) sides, there can be various other pairs of diagonals and consequently the area also is manifold."

चतुर्भुजे होकान्तरकोणावाक्रम्यान्तःप्रवेश्यमानौ भुजौ तत्संसक्तं स्वकर्णं संकोचयतः । इतरौ तु बहिःप्रसरन्तौ स्वकर्णं वर्धयतः । अत उक्तं तेष्वेव बाहुष्वपरौ च कर्णाविति । 171 com.

caturbhujē hyēkāntarakōṇāvākramyāntaḥpravēśyamānau bhujau tatsaṁsaktaṁ svakarṇaṁ saṁkōcayataḥ | itarau tu bahiḥprasarantau svakarṇaṁ vardhayataḥ | ata uktaṁ tēṣvēva bāhuṣvapárau ca karṇāviti |

"In a quadrilateral, when two opposite corners are so drawn as to bring the sides contiguous to them inwards, the diagonal joining them is shortened, while the other two corners bulge outwards and consequently their diagonal is lengthened. So it has been stated (just before) that with the same sides there can be other pairs of diagonals."

Further Bhāskara II says :

लंबयोः कर्णयोर्वैकमनिर्दिश्यापरं कथम् । पृच्छत्यनियतत्वेऽपि नियतं वापि तत्फलम् ॥
स पृच्छकः पिशाचो वा वक्ता वा नितरां ततः । यो न वेत्ति चतुर्बाहुक्षेत्रस्यानियतां स्थितिम् ॥ 171 com.

laṁbayōḥ karṇayōrvaikamanirdiśyāparaṁ katham |
pṛcchatyaniyatatvēऽpi niyataṁ cāpi tatphalam ||
sa pṛcchakaḥ piśācō vā vaktā vā nitarāṁ tataḥ |
yō na vētti caturbāhukṣētrasyāniyatāṁ sthitim ||

"Without specifying one of the altitudes or diagonals, how can any one ask to find the other of them and also the area, as these are truly indeterminate ? The questioner who does not know the indeterminate nature of a quadrilateral must be a blunderer ; still more is he , who answers such a problem."[17]

6.4 : After quoting Brahmagupta's theorem about the diagonals of a quadrilateral in verbatim, in verse 188, another criticism on it he has to make is that the calculation is unnecessarily complicated, when the diagonals can be more easily got from Brahmagupta's own method for forming the rational quadrilateral.[18]

अभीष्टजात्यद्वयबाहुकोटयः परस्परं कर्णहता भुजा इति ।
चतुर्भुजं यद्विषमं प्रकल्पितं श्रुतीस्तु तत्र त्रिभुजद्वयात्ततः ॥१८९॥
बाह्वोर्वधः कोटिवधेन युक् स्यादेका श्रुतिः कोटिभुजावधैक्यम् ।
अन्या लघौ सत्यपि साधनेऽस्मिन् पूर्वैः कृतं यद् गुरू तन्न विद्मः ॥१९०॥

abhīṣṭajātyadvayabāhukōṭayaḥ parasparaṁ karṇahatā bhujā iti |
caturbhujaṁ yadviṣamaṁ prakalpitaṁ śrutīstu tatra tribhujadvayāttataḥ ||189||
bāhvōrvadhaḥ kōṭivadhēna yuk syādēkā śrutiḥ kōṭibhujāvadhaikyam |
anyā laghau satyapi sādhanēऽsmin pūrvaiḥ kṛtaṁ yad gurū tanna vidmaḥ ||190||

"In the quadrilateral formed according to the rule : *abhīṣṭajātyadvaya* . . i.e. the sides (*bhujās*) and uprights (*koṭis*) of two right-angled triangles (*jyāyas*), being multiplied by the reciprocal hypotenuses (*karṇas*), become sides (of a quadrilateral), the diagonals (*sṛtis*) can be got from

the triangles themselves. The product of the *bhujās* combined with the product of the *koṭis* will be one diagonal and the other will be the sum of the reciprocal products of the *bhujās* and *koṭis*. When this easy method is available, I do not know why the earlier mathematicians should have adopted a more difficult method."

That is, if If e, f, g and p, q, r are the sides (integral or rational) of two (*jyātyas* i. e.) right triangles ('g' and 'r' being hypotenuses), from which the Brahmagupta quadrilateral is formed, then the diagonals will be $(ep + fq)$ and $(fp + eq)$. This is indeed an easier method for finding the mutually perpendicular diagonals of the Brahmagupta quadrilateral. But it is only in positions where the longest and shortest side face each other that the diagonals will be mutually perpendicular. For the other positions Bhāskara-II has a supplement in his notes:

अथ यदि पार्श्वभुजमुखयोर्व्यत्ययं कृत्वा न्यस्तं क्षेत्रम् तदा जात्यद्वयकर्णयोर्वधः द्वितीयकर्णः । 190. com.

atha yadi pārśvabhujamukhayōrvyatyayaṁ kṛtvā nyastaṁ kṣētraṁ tadā
jātyadvayakṛmayōrvadhaḥ dvitīyakarṇaḥ |

"If a flank and the face are interchanged, the product of the hypotenuses of the two *jyātyas* is the second diagonal."

Thus Bhāskara's method is easier in the case of quadrilaterals formed from known *jyātyas*. But *Brahmagupta's rule enables us to calculate the diagonals of any cyclic quadrilateral.* This, Bhāskara-II did not realize nor did he realize the cyclic character of Brahmagupta's quadrilateral. However **the idea of third diagonal in a quadrilateral is prompted here .**

VII. TRI- FORMS OF A CYCLIC QUADRILATERAL

7.1 : Third diagonal :

Interchange of two adjacent sides of a cyclic quadrilateral alters its shape and the length of one of its diagonal, there by getting a new diagonal, which is defined to be the third diagonal of the original quadrilateral.

7.2 : Theorem of three diagonals :

The geometry of three diagonals of a cyclic quadrilateral is discussed in greater detail by Nārāyaṇa Paṇḍita (1356 AD) in his *Gaṇita Kaumudī*. He says :

सर्व चतुर्बाहुना मुखस्य परिवर्तने यदा विहिते । कर्णस्तदा तृतीयः पर इति कर्णत्रयं भवति ॥ (G.K.iv-48)

Sarvacaturbāhunāṁ mukhasya parivartane yadā vihite |
Karṇastadā tṛtīyaḥ para iti karṇatrayaṁ bhavati ||

"When the top side and the flank side of any four sided figure are interchanged, we get a third diagonal called *para*. Thus there are three diagonals."[19]

7.3 : Tri- Forms Of A Cyclic Quadrilateral

Interchange of sides without changing the area is possible in a cyclic quadrilateral. The number of such a possible variations in the shape of the cyclic quadrilateral is equal to the number of arrangements of the four chords in the circle so as to form a quadrilateral. If we do not differentiate between the shapes of a rectilinear cyclic figure and its mirror image, then the number of such possible area preserving variations is equal to the number of permutations in which the chords can be arranged in a circular way irrespective of the order

in which they are taken (clock wise or anticlockwise) to form the desired figure. Obviously this number is equal to [1/2 (n-1)!].

In case of a quadrilateral it is [1/2 (4-1) ! = 3] three.

Hence for the given four sides, cyclic quadrilateral is of triform (or trirūpi)

VIII. MATHEMATICIANS OF THE ĀRYABHAṬA SCHOOL

8.1 : Parameśvara's conclusion[20]:

The mathematicians of the Āryabhaṭa school understood the scope and applicability of Brahmagupta's results clearly. Parameśvara (1360-1455) commenting on Brahmagupta's कर्णाश्रितभुजघातैक्य ... verse (Br.Sp.Si.xii-28) quoted by Bhāskara II (Lilavati Vs- 188) says :

यस्मिन् चतुर्भुजे एवं प्रकल्पितं कर्णद्वयं भवति तत्र
सर्वदोर्युतिदलमित्यादिना आनीतं क्षेत्रफलं च स्फुटं भवतीति ।
एतत्क्षेत्रफलान्न्यूनमेव कर्णान्तर कृत क्षेत्र फलानि भवन्ति ।

yasmin caturbhuje evaṃ prakalpitaṃ karṇa dvayaṃ bhavati tatra
sarvadoryutidala mityadinā ānītaṃ kṣetraphalaṃ ca sphutaṃ bhavatīti |
etatkṣetraphalānnyūnameva karṇāntara kṛta kṣetra phalāni bhavanti ||

"In quadrilaterals , the diagonals of which are calculated with the help of this formula, the area obtained by applying the formula *sarvadoryutidala* etc. i. e. $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ will be the exact area. The areas of quadrilaterals constructed with other diagonals will be less than this."

8.2 : Determinate Diagonals (Niyataśṛti or Niyata Karṇas) :

The discussion of the three determinate diagonals (*Niyata Karṇas)* as found in the *Kriyākramakarī* (pp. 348-351) is more subtle. It shows that in a cyclic quadrilateral more than three diagonals are not possible. He says :

चापयोगः षोढा स्थितोऽपि तज्जीवाः त्रिविधा एव स्युः ,
द्वयश्चापयोर्योग जीवाया एव इतर चापयोग जीवात्वात् ।
एता योगजीवाः चतुर्भुज क्षेत्रगताः कर्णाः ।
- com. on vs.189-190 (p.351)

Chāpayōgaḥ ṣōdhā sthitō 'pi tajjīvāḥ trividhā eva syuḥ
dvayascāpayōryōga jīvāyā eva itara cāpayōga jīvātvāt |
etā yōga jīvāḥ caturbhuja kṣetragatāḥ karṇāḥ |

The arguments given are substantially as follows: [21]

Let $\alpha, \beta, \gamma, \delta$ be the angular measures of the arcs corresponding to the sides a,b,c,d (respectively) of a cyclic quadrilateral. Now a sum of any two arcs can be made to define a diagonal. Hence there can be six cases. But because $\alpha + \beta + \gamma + \delta = 360^0$, there will be only three final possibilities [for example, if $(\alpha + \beta)$ defines one diagonal, $(\gamma + \delta)$ will define the same diagonal] . Hence, *three and only three diagonals are possible for four side-lengths of a cyclic quadrilateral.* The three determinate diagonals (*Niyata Karṇa)* of a cyclic quadrilateral can be calculated by Brahmagupta's Diagonal theorem (or *Karṇa Prameya).*Because of this, Jyeṣṭhadeva called the cyclic quadrilateral as *Niyataśṛti.*

In a square and a trapezium with three sides equal, these three diagonals are equal. In the isosceles trapezium and the rectangle, two diagonals are equal ; and in cyclic quadrilaterals of unequal sides all the three diagonals are unequal.

8.3 : Jyeṣṭhadeva's Emendation Of Bhāskara's Rule [22]

Jyeṣṭhadeva has emended the text of second half of the verse given by Bhāskara II for the area of a quadrilateral, so as to conform it with the Brahmagupta's rule and convey its correct meaning. It is as follows :

सर्वदोर्युतिदलं चतुःस्थितं बाहुभिर्विरहितं च तद्वधात् ।
मूलमत्र नियतश्रुतौ फलं त्यस्रबाहुजमपि स्फुटं भवेत् ॥

sarvadōryutidalaṁ catuḥsthitaṁ bāhubhirvirahitaṁ ca tadvadhāt |
mūlamatra niyataśrutau phalaṁ tyasrabāhujamapi sphuṭaṁ bhavēt ||

"Half the sum of all the sides is set down in four places ; and the sides are severally subtracted. The remainders being multiplied together, the square root of the product will be the exact area in quadrilaterals with determinate diagonals (*niyataśrti*) and so in the case of trilateral also."

IX. EPILOGUE

For continuation of the tradition and to pass on the knowledge acquired to coming generations without interruption, it is essential to interpret the available material in terms of contemporary terminology of the concerned science in an authentic and reader friendly manner. This is a small effort in that direction.

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5. TAS.p 90. fn.
6. TAS.p 91; Also see p.107.
7. HHM. II. pp. 238-239.
8. VDH, (2006). p.130-133
9. TAS. p.126.
10. Cf. *Triśatikā*.(*Tris*.) R.49. ; *Mahā-siddhānta* (MSi).xv-80(i) ; *Gaṇitakaumudī* (GK.)II.p.63.vs.53. or GK.iv.R-53.
11. Cf. *Brāhmasphuṭa siddhānta* (Br.Sp.Si.) xii-22 ; *Āryabhaṭīya Bhāṣya* by Bhāskara I (ABh).-ii-6(i) comm. P.57. ; *Gaṇita-Sāra-Saṅgraha* (GSS).vii-49 ; *Siddhānta- Śekhara* (Si.Se).xiii-29;
    a. Also see, GK.II.pp.45-46 vs.36-39(i), or GK.iv.R-36-39(i).
12. Cf. MSi.xv-86-87 ; GK.II.p.52.vs.43-45(i). or GK.iv.R.43-45(i).
13. TAS. p. 81.
14. TAS. p. 84.
15. Cf. Br.Sp.Si. xii-21(ii) ; PG.R.117 ; Tris.R.43(i). ; GSS.vii-50(i). ; MSi.xv-69 ; Si.Se.xiii-28 ; GK. II.pp.39-40, vs.33(ii)-34(i), or GK.iv.R-33(ii)-34(i).
16. HHM, II, pp.142-143.
17. TAS. pp. 87-88.
18. TAS. pp.95-96.
19. TAS. p.96.
20. TAS. pp.106-107.
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Japanese Approach to "Reaching Every Learner" Lesson Study and Problem-Solving Lesson[1]

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Introduction

The theme of Golden Jubilee Conference of the AMTI is "Reaching Every Learner". I have been thinking that it may relate to the three stratums: Intended, Implemented, and Attained Curriculum (Husen 1967). For an international perspective, people tend to concern Attained Curriculum, such as results from PISA or TIMSS, or Intended Curriculum such as new curriculum in the US: Common Core State Standards. However, among three stratums, Implemented Curriculum seems to be the most critical factor relating to the theme: "Reaching Every Learner. The TIMSS 1995 Video Study was the first attempt to focus on this stratum from an international perspective, concentrating on the eighth grade mathematics lessons in USA, Germany, and Japan. The Teaching Gap (Stigler and Hiebert 1999) summarized the results from the TIMSS video study and identified some similarities and differences among three countries. In addition, some uniqueness of Japanese mathematics lessons were revealed. For example, the percentage of lessons that included student-presented (teacher-presented percentage is put in parentheses) alternative solution methods is USA: 8% (19%), Germany: 14% (12%), and Japan: 42% (7%); (p.69).

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Behind differences there could be some important educational factors that need to be considered. Therefore, Chapter seven of The Teaching Gap in particular, titled "Japan's approach to the improvement of classroom teaching", which is based on Yoshidas (1999) doctoral dissertation, now available in book form Fernandez and Yoshida (2004), has attracted peoples attention and provoked enormous interest in Lesson Study. The authors of The Teaching Gap characterized Lesson Study as a cultural activity in Japan and as a powerful teacher professional development activity. Since then, many mathematics teachers and teacher educators have been trying to implement Lesson Study in their countries, and many books and research papers have been written on various aspects of Lesson Study (Lewis 2002; Lewis and Hurd 2009, 2011; Hart, Alston and Murata 2011; Doig and Groves 2011; Department for Children, Schools and Families 2008; White and Lim 2008; Ono and Ferreira 2010), and the typical lesson flow of Japanese mathematics lesson, that is, the structured problem solving lesson as Stigler & Hiebert (1999) described (Perry & Hurd 2009; Takahashi 2006, 2008;Takahashi, Lewis and Perry 2013).

But some aspects of Lesson Study that may be taken for granted by Japanese teachers seem not to be well understood outside Japan. For example, unlike many Western initiatives, Lesson Study in Japan is neither funded nor obligatory. It is essentially school-based and organized by teachers themselves. For Lesson Study outside of Japan, there needs to be a more clearly defined structure of Japanese model of Lesson Study (Fujii 2013) and precisely elicit teachers beliefs and attitudes behind the process of Lesson Study.

For considering the theme: "Reaching Every Learner from

Japanese perspective, I try to discuss why it is inevitable to unify Lesson Study with the structured problem solving lesson in mathematics. For this purpose, I introduce three levels of teaching, and then describe in detail the process of teaching/learning mathematics through problem-solving and Lesson Study in Japan.

Putting the Conference Theme into Practice: Three Levels of Teaching

As a way to uncover the differences in teacher expertise, let me apply the professor Sugiyamas three levels of teaching, Level 1, Level 2, and Level 3: *Level 1: Just to explain how to do it.*
The teachers who can show important basic ideas of mathematics such as facts, concepts, and procedures.
Level 2: To explain how to do it and why.
The teachers who can explain meanings and reasons for important basic ideas of mathematics in order for students to understand them.
Level 3: To let the students discover for themselves how to do it and why.
The teachers who can provide students opportunities to discover and understand these basic ideas, and support their learning in order for students to become independent learners (Sugiyama 2008).

Prof. Sugiyamas typical example to distinguish Level 1 and Level 2 is fraction divided by fraction. Richard Skemp (1976) used the same example to distinguish instrumental understanding which is equivalent to Level 1 and relational

understanding which is to Level 2. So how about Level 3? Looking from the surface, the Level 3 seems to be the easiest one for the teacher because students will be doing most of the work. But if you will look at it deeply, there are underlying prerequisite knowledge and competency teaches must possess. For example, teachers should know what students in a particular grade know that they can use to tackle the task fruitfully. This way, he/she can determine where to take off using self-discovery on the part of the students.

For becoming the Level 3 teacher, besides Shulman(1986) pedagogical content knowledge(PCK), I am in a way suggesting three additional categories of the knowledge base. One is knowledge of organizing lessons as problem solving approach. The other is knowledge of Lesson Study as a system. And the last one is recognition of the value and methodology of Kyozai-kenkyu.

The meaning of the Japanese word Kyozai-kenkyu is to research or study on teaching materials in terms of mathematical and educational point of view as well as from students point of view (Watanabe, Takahashi and Yoshida 2008). We give careful consideration to typical conceptions and misconceptions students may have, because these would surely come out at problem solving processes. From Japanese cultural perspectives, therefore in order to improve the quality of mathematics education in terms of Reaching Every Learner, we need to implement the structured problem solving lesson based on Kyozai-kenkyu.

The Level 3 Teaching Crystallizes in the Structured Problem Solving Lesson

Researchers from outside Japan have noted patterns in Japanese mathematics lessons. Jerry P. Becker et.al. (1990) identified eight components in typical mathematics lessons; Stigler and Hiebert (1999) identified five. But their points of view are those of observers; Japanese teachers do not usually think about the structure of their lessons in the same way. For instance, the first component of Stigler and Hiebert (1999), reviewing the previous lesson, is not an important activity from the Japanese teachers point of view. What we do consciously in problem-solving lesson in mathematics are the following four activities:

1. Presenting problem for the day (from the childrens point of view, it could beunderstanding the problem). (5 to 10 minutes)
2. Problem solving by the students. (10 to 20 minutes)
3. Comparing and discussing (Neriage). (10 to 20 minutes)
4. Summing up by the teacher (Matome). (5 minutes)

These four activities or components are a kind of framework or form of the lesson. What teacher actually does in each framework are as followed:

Presenting problem for the day

The Japanese structured problem solving lesson is not Show and Practice (Lappan and Phillips 2009). Some people may interpret presenting problem means detailed explanation

with the procedures involved in solving the problem or demonstration on how to solve the task. In fact, in the Teaching Gap the authors described that the (Japanese) teacher presents a problem to the students without first demonstrating how to solve the problem. And We realized that U.S. teachers almost never do this, and how we saw that a feature we hardly notice before is perhaps one of the most important features of U.S. lessons-that the teacher almost always demonstrates a procedure for solving problems before assigning them to students.(p.77) In Japan, presenting problem means to make students understand the context embedded in the task and corresponding mathematical condition that would be used to solve the task. Japanese educators distinguish between teaching how to solve the task and teaching mathematics through solving the task. This is why most structured problem solving lessons focus on a single task. If chosen well, a single task allows for the important new mathematical ideas to emerge in the discussion, and additional tasks are unnecessary. Thus we have the following principles for an ideal task (Fujii 2015):

a. It is appropriate and mathematically valuable in terms of the aims of the lesson;

b. It interests the students;

c. It is at the appropriate level of difficulty;

d. It can be solved in several ways;

e. It has a potential to elicit valuable basic wisdom.

Usually, only one task is given in a lesson. You may feel it is rather strange to give only one problem in a mathematics

lesson but this is our way of doing it and we carefully choose and present the task in terms of the aims of the lesson. We put the conscious effort in choosing the numbers used in the problem. For example the topic: two-digit number(less than 20) minus a single-digit number with regrouping or borrowing from first grade textbook, the first problem is 13 - 9 or 12-9 among 36 problems (Fujii 2015). These numbers are considered as the quasi-variables (Fujii and Stephens 2001, 2002, 2008).

Problem solving by the students

While students are solving the problem by themselves, teacher goes around among students checking and monitoring their work. In Japanese we term it as kikan-jyunshi. This means purposeful scanning or monitoring. Almost all teachers in other countries behave similarly but what is consciously being done may differ from one country to another. To prove this point, Hino(2003) identified the result from the Learners Perspective Study to distinguish the initial effort between Australian teacher and Japanese teacher. An Australian Math teacher was interviewed and said as follows:

Ms M Well, I have to make sure that every-one's involved and every-one's participating in the lesson I guess – that I'm not excluding anyone – that um – that I'm encouraging everyone to have a go

It seems that she is trying to consult with every student individually. So what do Japanese teachers consciously do during kikian-jyunshi?

a. With the teacher looking at the seat plan, he/she identifies which student is doing which types of solution that were anticipated beforehand. (The anticipated solutions including typical incorrect solutions should be created while teachers prepare their lesson plans.)

b. To capture the general tendency of the class.

c. To give a hint to slow learners if necessary and to ask fast learners to come with another way of solving and so on.

d. To plan what he/she should do at the comparison and discussion period.

One of the most difficult parts is to make anticipated solutions beforehand – this is particularly difficult for novice teachers. I think Japanese teachers teaching ability improves as they try to prepare all possible solutions. Or more generally speaking, planning lesson activity is a critical factor relating to the quality of mathematics teaching and to put reaching every learner into reality. With this, any teachers knowledge for that matter and also for students cognitive nature becomes richer.

During kikan-jyunshi, If you find out the solutions beyond your expectations, you may ask what the student did to come with that solution in a detailed way and record it. This is also another way of enriching the teachers knowledge for teaching mathematics. Besides individual solutions, it is also important if the teacher will look into the general tendency of the class.

While you are doing kikan-jyunshi, you may have to decide which student will present first and then where should the climax of the lesson be. In short, while doing kikan-jyunshi what the teacher does is to collect information and make

scenarios for comparison-discussion activity. Usually, student-teachers or novice teachers find kikan-jyunshi extremely difficult for them to implement in lessons.

Comparing and discussing (Neriage) The third phase, called neriage in Japanese, assumes that students will arrive at different solution methods and focuses on a comparison and discussion of those different solution methods. Therefore teachers discuss the plausibility of the anticipated student solutions listed in the lesson plan. At Neriage period, solutions by the students are presented usually from "nave to "sophisticated ones at the blackboard. Therefore, the teacher should be careful because certain students may realize why they are always the first person to present. In Japan, in order to compare and contrast students solutions the teacher may carefully plan the blackboard writing. Blackboard is used by Japanese teachers to organize both the thought process and result. Therefore, Japanese teachers had been reluctant to use an OHP. An OHP is not suitable for showing the accumulated process of thought. Comparison-discussion period(Neriage) is also difficult for novice teachers because this activity is closely connected to kikan-jyunshi activity. The novice teachers could give positive comment on each student but struggle to elicit, clarify something the students cannot see. Sometimes, the students cannot distinguish between two similar solutions but sometimes the teacher has to make comments that these two solutions are significantly different from each other or the opposite thing that is in the surface they seem to be different but they are actually the same.

Comparison-discussion period(Neriage) is a place to foster

mathematical way of thinking. Mathematical way of thinking cannot be demonstrated by using only the definition but rather through problem solving activities. It is also a place to realize to think mathematically and eventually we hope that students will become intellectually independent persons. This is the final goal for Level 3 lesson. The lesson is education for human beings to grow together mathematically. Therefore, we have to see the students potential as high as possible rather than underestimate them.

Sometimes the comparison-discussion activity may appear to be implemented well but in reality that is not the case. There would be a one-to-many correspondent between the teacher and the students. Takahashi (2008) described this phenomenon as show-and-tell. The heart of the comparison-discussion activity is reflective discourse (Doig, Groves and Fujii 2011). Although rather a classic citation, the comparison-discussion activity is a place to realize Vygotskys zone of proximal development.

Summing up (Matome)

Fujii, et al (1998) identified the tendency that American teachers do not do summing up. Without summing up by teachers, lesson would be over with individual students satisfaction. However lesson is better to be finished with not only individual satisfaction but also mathematical satisfaction. "Summing up" is done only for a short period but it is critically important. Students may listen carefully what the teacher says as final words. If the teacher conclude lesson with a procedure oriented comment, students may feel procedure knowledge is most important in Mathematics. The teacher should mention how students construct their ideas by using their former or

previous knowledge. How we think is most highly valued. At summing up activity, the teachers value system is evident.

Improving Quality of Mathematics Education through Lesson Study

Lesson Study is an approach to teacher professional development that differs sharply from the professional development practices common in other countries. Lynn Liptak contrasted Lesson Study with the traditional professional development as practiced in the United States as below Table 1 (cited from Lewis (2002)):

Lesson Study begins with a question, not with an answer

Traditional Professional Development	**Lesson Study**
Begins with answer	Begins with question
Driven by outside "expert	Driven by participants
Communication flow: trainer to teachers	Communication flow: among teachers
Hierarchical relations between trainer and teachers	Reciprocal relations among learners
Research informs practice	Practice is research

Table 1: Constructing Views of Professional Development by Lynn Liptak, cited from Lewis(2002)

prepared by someone else. Identifying this question, which becomes the research theme for Lesson Study, is the first step in the process which can be illustrated as in Figure 1 (Fujii 2014,

p.3). The research theme is developed through consideration of the reality of students current state vis--vis educational or long term goals for their learning and development.

This diagram, with five steps, contrasts with similar diagrams in other publications that have four steps (e.g. Lewis 2002; Lewis and Hurd 2011). These five steps more accurately portray the reality of Japanese teachers Lesson Study activity. Borrowing from Lewiss descriptions (Lewis 2002, Lewis and Hurd 2011), each step can be summarized as follows:

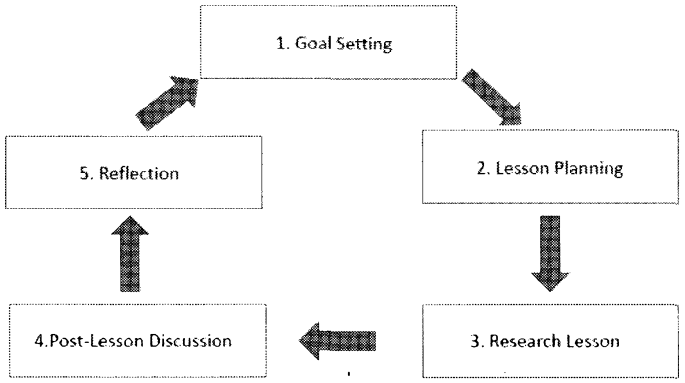

Figure 1: The Process of Lesson Study

1. Goal setting: Consider long-term goals for student learning and development. Identify gaps between these long-term goals and current reality, and then formulate the research theme.

2. Lesson Planning: Collaboratively plan a research lesson designed to bring to life these goals. Write and revise a detail instruction plan that includes: Long-term goals, Anticipated student thinking, Data collection, Model of

leaning trajectory, Rationale for chosen approach, etc.

3. Conduct the research lesson: One team member teaches the research lesson, others observe and collect data.

4. Conduct the post-lesson discussion: In a formal lesson colloquium, share data from the lesson to illuminate student learning, discrepancies in content, lesson, and unit design, and broader issues in teaching-learning.

5. Reflect: Document the cycle to consolidate and carry forward learnings as well as new questions into the next cycle of Lesson Study. Write report or bulletin that includes lesson plan, student data, and reflections on what was leaned.

Outside of Japan the third and fourth steps in the Lesson Study cycle - being most visible - attract peoples attention. This leads to a typical misconception of Lesson Study: that the research lesson and post-lesson discussion are Lesson Study. Indeed, Lesson Study is sometimes introduced as an open lesson by a veteran teacher jumping in to another teachers classroom (Takahashi 2013, p.84). But each step in the Lesson Study cycle is closely related to the others, and the third and fourth steps are especially based on the first and second steps. My talk highlights on the third, research lesson in Lesson Study cycle. The research lesson in mathematics usually forms the structured problem solving lesson.

In both pre-service and in-service teacher development programs, Lesson Study plays an important role in supporting teacher growth in Japan. Lecture and workshop type of professional development, known as phase 1 professional

development (Takahashi 2011), could be effective in increasing a teachers knowledge for teaching mathematics. On the other hand, Lesson Study, known as phase 2 professional developments, focuses on developing expertise for teaching mathematics. This includes the expertise needed to develop lessons for particular students with educational aims (Step 1 in Fig.2), to use various questioning techniques, to design and implement formative assessments, to anticipate student responses to questions and tasks (Step 2), and to make purposeful observations during class (Step 3). For teachers to develop such expertise, they need opportunities to plan lessons carefully(Step 2), to teach the lesson based on the plan(Step 3), and to reflect upon the teaching and learning based on careful observation(Step 4 and Step 5). Japanese teachers and educators obtain these experiences through Lesson Study.

Unifying Lesson Study with the structured problem solving lesson in mathematics

When conducting the structured problem solving lesson at Step 3 in the Lesson study cycle, the aim of the lesson is to foster students mathematical way of thinking and habits of mind. However, thinking itself and thinking processes are invisible. Therefore, we cant teach how to think directly. Only thing we can do is indirectly suggesting how to think mathematically. Therefore, it is extremely difficult for a teacher to teach how to think. What the teacher can do is to anticipate students responses beforehand at Step 2 that is the second component of Lesson Study: lesson planning. And at Step 3: the research

lesson, the teacher needs to grab a tail of thought when it comes to be visible, then the teacher uses that tail as a concrete example to teach or suggest how to think mathematically in general. There we need the research lesson as an open situation with many observers eyes. At the research lesson, teacher colleagues observe what the teacher is doing in terms of dealing with students invisible thinking processes. And then they discuss or evaluate the teachers behaviors whether those were appropriate or not at the lesson, that is Step 4 in the cycle. And finally participating teachers reflections are written and published as an annual report from school. Each step in Lesson Study cycle is closely related. Lesson Study is a system for teacher to learn: how to teach students in order for them to become independent thinkers.

One of the critical features of problem-solving lesson in the context of Lesson Study is that task used in the lesson is to be evaluated at the post-lesson discussion. The task is not judged based on some abstract determination about whether it is good or not for teaching a certain skill or concept, but based on concrete evidences from the research lesson about how the students responded to it. That evaluation often goes beyond the specific content of the lesson. But teachers also discuss more general issues. At the post-lesson discussion, the final commentator also addresses broader educational values. For example, the final commentator appeared in the The Teaching Gap explicitly addressed broad educational values in the very beginning of his comments. He urged teachers to think carefully about what were the most important skills for living that students should be learning from their mathematics instruction. (p.182) Using as an example the formula for finding the area

of a trapezoid, he said that "teachers should help students realize that moving from complicated to more simple forms is a convenient and a clever thing to do. (p. 183) This is an example of how Japanese Lesson Study incorporates educational values. This notion is related to the problem-solving type of lesson. A common misconception about such lessons is that the solving the task is the main point. Such misconception leads to a focus on goals such as students can do X or "students understand X. But a problem-solving lesson is not just about finding the solution to a problem. It is well and good that students can do X, but X should contain some value, and what that value is needs to be considered. To identify the educational values, the final commentator urged the teachers to do Kyozai-kenkyu. The goal of Kyozai-kenkyu is to develop tasks and lessons that bring broad educational values to life in the classroom.

Conclusion

Lesson Study can be described as a kind of teacher professional development. However Lynn Liptak(2002) clearly declared the difference between Traditional Professional Development and Lesson Study. According to Lynn Liptak, Lesson Study begins with a question, not with an answer prepared by someone else – this character is the most critical nature of Lesson Study. Considering the nature of Lesson Study that begins with question, the first step of Lesson Study is to construct the research theme. When teachers decide the research theme, they consider the reality of students and educational goals, or long term goals of students learning and development. This process needs to be the necessary step for improving the quality of mathematics education for each country.

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What is the Theory of Majorisation? [1]

Rajendra Bhatia

Look at the following 3-vectors:

$$w = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \quad x = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \quad y = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}, \quad z = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}. \tag{1}$$

What is a feature common to all of them?

The coordinates of each of them sum up to 9, and are nonnegative. Suppose these coordinates represent a division of nine identical objects into three persons. Then which of them represents a fairer division? Clearly z is the most unfair, as one person corners everything, and w the most fair. How do x and y compare with others on this scale of fairness?

The theory of majorisation is a subject that provides a framework for such questions. Let $\mathbb{R}^n$ be the vector space consisting of n-tuples $x = (x_1, \ldots, x_n)$ of real numbers. Let $x^\downarrow = (x_1^\downarrow, \ldots, x_n^\downarrow)$ be the vector whose coordinates are $x_1, \ldots, x_n$ rearranged in decreasing order. Thus if $x = (3, 4, 2)$, then $x^\downarrow = (4, 3, 2)$. Notice that if $x = (2, 3, 4)$, then too $x^\downarrow = (4, 3, 2)$. We say that x is majorised by y, and write this as $x \prec y$, if the following relations are satisfied:

$$\begin{aligned} x_1^\downarrow &\leq y_1^\downarrow \\ x_1^\downarrow + x_2^\downarrow &\leq y_1^\downarrow + y_2^\downarrow \\ &\vdots \\ x_1^\downarrow + x_2^\downarrow + \cdots + x_{n-1}^\downarrow &\leq y_1^\downarrow + y_2^\downarrow + \cdots + y_{n-1}^\downarrow \\ x_1^\downarrow + x_2^\downarrow + \cdots + x_{n-1}^\downarrow &= y_1^\downarrow + y_2^\downarrow + \cdots + y_{n-1}^\downarrow. \end{aligned} \tag{2}$$

Suppose a sum s is distributed among n individuals labelled as $1, 2, \ldots, n$. In one scheme these individuals receive $y_1, \ldots, y_n$, respectively, and in another they receive $x_1, \ldots, x_n$, respectively.Then the system of inequalities (2) says that the maximum share of any individual in the second scheme is not bigger than that in the first scheme. The top two shares in the second scheme together are no bigger than that in the first scheme; and so on. The last relation in (2) is an equality as both sides have to sum up to s. Thus the relation $x \prec y$ between non negative vectors may be interpreted to mean that x represents a more equitable distribution of wealth than y.

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1 Presidential Address
50th Annual Conference, Association of Mathematics Teachers of India (AMTI)

A vector $p = (p_1, \ldots, p_n)$ is said to be a probability vector, if $p_j \geq 0$ for all $1 \leq j \leq n$, and $\sum p_j = 1$. The number p_j represents the probability of the j th event among uncertain events $1, 2, \ldots, n$. For any such vector we have

$$(\tfrac{1}{n}, \tfrac{1}{n}, \ldots, \tfrac{1}{n}) \prec (p_1, \ldots, p_n) \prec (1, 0, \ldots, 0). \tag{3}$$

The probability vector $(1, 0, \ldots, 0)$ represents the situation that the first event occurs surely. So there is no uncertainty in this case. On the other hand, the vector $(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$ represents the case when all the n outcomes are equally probable. So the uncertainty is maximum in this case. Thus the relation $\prec$ is useful in comparing uncertainties in statistical experiments.

In another context, a vector $x = (x_1, \ldots, x_n)$ could represent the "state" of a physical system. Then $x \prec y$ is a way of saying that a system in state x is "more scattered" than one in state y.

Thus it is of interest to study what properties the relation $x \prec y$ has, and how they can be used.

The space $\mathbb{R}^n$ consists of ordered n-tuples of real numbers. Let us say two elements x and y of $\mathbb{R}^n$ are equivalent, if the coordinates of x are just a permutation of the coordinates of y. Thus, for example in $\mathbb{R}^3$ we say that $(5, -1, 0), (-1, 0, 5)$ and $(0, 5, -1)$ are all equivalent. Let us donate by $\mathbb{R}^n_{sym}$ the space of equivalence classes so obtained. (In general, there are $n!$ points in $\mathbb{R}^n$ in each equivalence class)

Then $\prec$ defines a partial order on the space $\mathbb{R}^n_{sym}$. (You may verify this)

Let us try to understand what the relation $x \prec y$ means when $n = 2$. Given a vector $y = (y_1, y_2)$ what are all vectors $x = (x_1, x_2)$ that are majorised by y? For concreteness let us take $y = (3, 1)$. Then all vectors x such that $x \prec y$ must obey the following relations.

(i) $x_1 + x_2 = 4$;

(ii) if $(x_1, x_2) \prec y$, then $(x_2, x_1) \prec y$;

(iii) max $(x_1, x_2) \leq 3$, min $(x_1, x_2) \geq 1$.

Use these considerations to see that $x \prec y$ if and only if x lies on the line segment joining the points (3,1) and (1, 3). Show that this is true in general: if x and y are any two points in $\mathbb{R}^2$ then $x \prec y$ if only if x is on the line segment joining (y_1, y_2) and (y_2, y_1).

This can me formulated in an algebraic language. A point (x_1, x_2) is on the line segment joining (y_1, y_2) and (y_2, y_1) if and only if there is a number

t between 0 and 1 such that

$$x_1 = (1-t)y_1 + ty_2, \qquad x_2 = ty_1 + (1-t)y_2$$

(Prove this.) These two equations can be combined into one matrix-vector equation

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1-t & t \\ t & 1-t \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \tag{4}$$

If S is the 2×2 matrix

$$S = \begin{bmatrix} 1-t & t \\ t & 1-t \end{bmatrix}, \tag{5}$$

then the equation (4) can be written as

$$x = Sy. \tag{6}$$

Can we learn enough from these considerations to guess how the relation $x \prec y$ may be characterised in geometric and algebraic terms in the case of $\mathbb{R}^n$ for $n > 2$?

A set K in $\mathbb{R}^n$ is said to be convex if whenever x and y are two points in K then the line segment L joining x and y lies entirely inside K. (In other words, if $z = (1-t)x + ty$, for some $0 \leq t \leq 1$, then z is in K.) In the plane $\mathbb{R}^2$ every straight line, triangle, rectangle, circle, and ellipse bounds a convex region. On the other hand the shaded regions

are not convex. In the space $\mathbb{R}^3$, a cube, a tetrahedron and a sphere, all bound a convex region, but a cycle tube does not.

An $n \times n$ matrix $S = [s_{ij}]$ is said to be doubly stochastic if

(i) $s_{ij} \geq 0$ for all $1 \leq i, j \leq n$;

(ii) $\sum_{j=1}^{n} s_{ij} = 1$ for all $1 \leq i \leq n$;

(iii) $\sum_{i=1}^{n} s_{ij} = 1$ for all $1 \leq j \leq n$.

The first condition says that all entries of S are nonnegative, the second says that the sum of entries of each row of S is equal to 1, and the third says that the sum of entries of each column of S is also equal to 1. Notice that the matrix S in (5) is doubly stochastic. What we have discussed above for $\mathbb{R}^2$ is subsumed in the following famous theorem, generally called the Hardy-Littlewood- Polya Theorem:

Theorem Let x and y be any two points of $\mathbb{R}^n$. Then the following three conditions are equivalent to each other

(i) $x \prec y$;

(ii) x lies in the convex polyhedron whose vertices are the points y_σ obtained by permuting the coordinates of y;

(iii) $x = Sy$ for some $n \times n$ doubly stochastic matrix S.

You should see how this generalises what we discovered (and proved) for majorisation in $\mathbb{R}^2$. Of course, the case $n > 2$ is not as easy. I will return to this theme in my next address to you!

If I have succeeded in arousing your curiosity you could peep into (but not necessarily read) two books. *Inequalities* by G.H. Hardy, J.E. Littlewood and G.Polya published in 1934 is one of the classics in mathematics. It devotes several of its chapters to majorisation. Entirely devoted to the single theme of majorisation is the 569-page book *Inequalities: Theory of Majorisation and Its Applications* by A.W. Marshall and I Olkin, published in 1979.

Each of the three authors of the first book has written other books that I recommend to you for reading. These are G.H. Hardy, *A Mathematician's Apology*, J.E. Littlewood, *A Mathematician's Miscellany* and G. Polya, *How to solve it*. Hardy's book is a reflection on the life of a mathematician, and on the creative process in general. Littlewood's is a collection of short essays and comments on mathematics, mathematicians, and others. Polya tells his readeres about the art of solving problems. All three make interesting reading for everyone.

Association Activities

We thank the outgoing President Prof. Rajendra Bhatia for his inspirational guidance and support to our activities. We

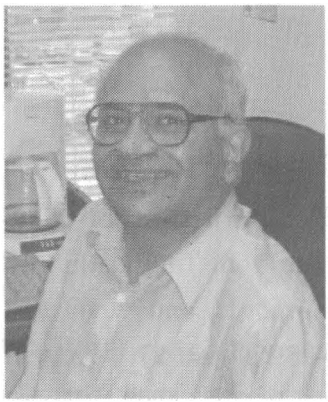

Figure 1: Prof R Bhatia

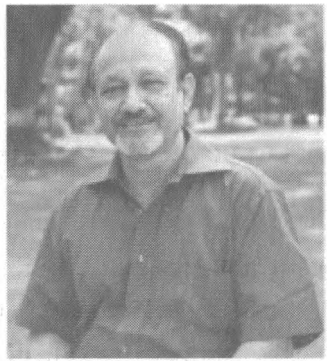

Figure 2: Prof Inder K.Rana

welcome our new President, Prof. Inder K Rana, Department of Mathematics, IIT, Powai, Mumbai 400076.

1. During 2016 May workshops the number attended are given below.

Level	No	Dates
Primary	71	May 4th to 7th
Sub-Junior	138	May 11th to 14th
Junior	84	May 18th to 21st
Teachers	32	May 25th to 28th

2. On library : We have about 10000 books being arranged in the library for use by prospective teachers, students and visitors. Some of the books may also help researchers. Interested persons may contact us to know the working hours.

3. NMTC

Level	NMTC 48	NMTC 47
Number of schools	795	757
Open contestants	344	300
Total registered	141399	124772

The screening test was conducted on 27th August and final exam on 22nd October.

4. The RMO 2016 was scheduled on 16th October 2016, between 1 pm and 4 pm. Students from Tamilnadu and Puduchery studying in classes 9 to 11 and who were among the best 10% in NMTC 48 held on 27th August 2016 were eligible to participate in the exam. There was also a pre RMO for such children and those selected wrote the RMO. The exam was conducted at 5 centers – Chennai, Coimbatore, Madurai, Tanjavur and Trichy. About 30 to be selected from among them to write INMO on 15th January 2017. Details will be communicated to the qualified students. INMO will be held in Chennai only.

5. We are planning to start concept clarification and Olympiad orientation classes through the internet. Students can post their doubts on our webpage and experts will answer their queries. Interested aspirants may register by sending their details by filling the proforma given in our website. Last date for registration is 1st February, 2017.

Professor K S RAMACHANDRAN

We regret to announce the demise of Professor K S Ramachandran, affectionately called KSR, on July 2nd 2016. A retired professor of Mathematics from Vivekananda College, Chennai, he was a Sanskrit scholar, Carnatic music connoisseur and an affectionate and wonderful person. He was ready to help any student of Mathematics and has done innumerable help to countless people. He was very unassuming and easily approachable, and inspired his students to excel in Mathematics. He is a revered teacher and his students fondly remember his efficient way of teaching. He was an inspiration to Mathematics teachers as well and will be missed by all of us.

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